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The Florentine maestri d'abaco Piermaria (1457–1508) and Filippo (Maria) (1468–1518) di Calandro Calandri and their works

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The Florentine *maestri d'abaco* Piermaria (1457–1508) and Filippo (Maria) (1468–1518) di Calandro Calandri and their works

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1 Introduction

So far known, five mathematical incunabula in two Italian varieties[1] (further editions not counted) survived.[2] Four of them written in the Venetian variety (and, unless anonymous, their authors) were already examined in modern English contributions.[3] In contrast to that, mostly Italian researchers payed attention (in their language) to the Florentine master arithmeticians (*maestri d'abaco* or *abacisti*) Piermaria (Florence 1457-03-06 – 1508-02-02 Florence) and his younger brother Filippo Maria (Florence 1468-01-07 – 1518-04-13 Rome) di Calandro Calandri and their works. The name was often spelled *Chalandri* according to the Tuscan fricative pronunciation (*gorgia toscana*) of [k] as [h]. The textbook *De arimethrica opusculum* (editions 1492 and 1518) and five manuscripts, all of them in the Florentine variety, are extant.[4]

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1 Venetian and Florentine. Today's linguists use the expressions "variety" or "regional language" instead of "dialect" due to its negative connotation. The standardized modern Italian language (based on the Tuscan variety) was only created in the 19th century with Alessandro Manzoni's (1785–1873) novel *I promessi sposi* (*The betrothed*, 1827).

2 UCatInc (Union Catalogue of Incunabula – Gesamtkatalog der Wiegendrucke), ISTC (Incunabula Short Title Catalogue), Egmond 1980, Hoock 1991.

3 Cf. 'Treviso Arithmetic' *Arte dell'abbaco* 1478 in Smith 1924 and Swetz 1987; 'Venezia Arithmetic' *Algurisimo* 1476/80 in Karpinsky 1929; Piero Borghi (Venezia ca. 1414 – ca. 1484/91 Venezia), *Aritmetica mercantile* 1484, 1488 and 1491 in Smith 1926; Luca Pacioli (Sansepolcro ca. 1445–1514/17 Rome), *Summa de arithmetica, geometria, proportioni e proportionalità* 1491 e.g. in Cajori 1924 and Geijsbeek 1914; cf. Holl 2024.

4 Tropfke 1980 contains numerous mathematical references, but wrong biographic and incomplete bibliographic data (p. 672); Smith 1908 mentions the textbook with a brief description and some illustrations (p. 47–49). But there is no systematic investigation about the Calandri brothers in English whereas Elisabetta Ulivi (2013) presents detailed research on the Florentine mathematicians in the

In this paper, we intend to summarize the Italian secondary literature about these mathematicians and to add some comparative results regarding their works.

2 Florence in the 2nd half of the 15th century

The lives of the master arithmeticians Calandro and his two sons Piermaria and Filippo Maria fall into the period of the **Italian Renaissance** (*rinascità* or *rinascimento*, rebirth of classical antiquity) that led to flourishing economy, science, art and culture, initially in northern Italy and eventually throughout Italy and Europe. The starting point was Florence, also due to the high level of industry, extensive trade and an elaborate financial and banking system.

Florence was a republic, power lay within a small class of (male!) merchants, bankers, manufacturers, craftsmen and academics who were organized in guilds (*arti* 'arts'). The city councils were recruited from this ruling class, chosen by lot (!), had very short terms of office and were subject to a sophisticated system of separation of powers. Behind the scenes, politics consisted of the struggle for power between few rich families and their followers: until 1434, the long-established Albizzi (as the party of the rich) were mostly able to prevail, but from 1201 onwards, the Medici family came into play, becoming increasingly wealthy and influential.

Cosimo de' Medici (1389–1464, known as *Il Vecchio*, 'the Elder') was able to consolidate his power in 1434 after a triumphant return from a year's exile, although he himself only held an office for a very short time: he installed his followers in important positions, had a wide network at home and abroad and eventually became banker of the Papal States. Cosimo was interested in a balanced relationship between the Italian powers of Milan, Venice, Florence, Naples and the Papal States (which was certainly beneficial for his business), and in foreign policy, he always worked to keep peace. Cosimo's lifestyle was modest and bourgeois, but he was extremely generous as a patron of important artists (including the painters Fra Angelico and Fra Filippo Lippi, the architects Filippo Brunelleschi and Michelozzo and the sculptor Donatello) and humanists. He had churches and monasteries built and furnished and was a passionate collector of valuable books.

One year after Cosimo's death in 1464, Florence awarded him the official honorary title of *pater patriae* ('father of the fatherland'), and his power passed smoothly to his son, the rather colourless **Piero** (1416–1469, called *Il Gottoso* after the gout that the Medici family suffered from).

The climax of this culturally extraordinarily splendid era, however, was only reached with Piero's son **Lorenzo** (1449–1492), called *Il Magnifico*, 'the Magnificent'). Lorenzo had received a comprehensive humanistic education in languages, philosophy, finance and warfare. Like his grandfather Cosimo, he ruled Florence through intermediaries. The Pazzi conspiracy on Easter Sunday 1478 is famous: Lorenzo not only survived it, but emerged from it even stronger. In foreign policy, Lorenzo, like his grandfather Cosimo, pursued a policy of balance and

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early modern times, and Gino Arrighi (1969, 1974, 1976 (only in parts)) and Daniela Santini (1982) edited four of the Calandri manuscripts.

peace. Even when he came into open conflict with the Papal States under Sixtus IV and the Kingdom of Naples, he achieved a peace treaty through extremely courageous and smart negotiations and united the Italian powers of Rome, Florence, Naples, Milan and Venice (also against the invasion by Mehmed II, the conqueror of Constantinople).
Lorenzo's banking business seems to have been less important to him; at one point, he was close to bankruptcy. Lorenzo was called *Il Magnifico* due to his extremely generous patronage of art (including Sandro Botticelli and Michelangelo Buonarroti) and philosophy (Giovanni Pico della Mirandola, Angelo Poliziano, Marsilio Ficino). He himself was also an outstanding poet. The Magnifico had married Clarice Orsini from one of the most influential Roman families; seven of their numerous children survived, including Giovanni (Florence 1475–1521 Rome), the later Pope Leo X, and **Giuliano** (Florence 1479–1516 Fiesole), the dedication addressee of Filippo Calandri's textbook *De arimethrica opusculum* (1491).[5]
After Lorenzo's early death, he was succeeded by his son Piero (1472–1553, known as *Lo Sfortunato*, 'the Unfortunate'). As he had made bad decisions, he was already expelled from Florence in 1494. Power was taken by **Girolamo Savonarola** (1452–1498), a Dominican monk and opponent of the Medici, who established a kind of theocracy in Florence between 1494 and 1498 through flaming sermons with end-time visions. Famous is the "Bonfire of the Vanities", a burning of the Florentines' luxury goods, but also of books and art, as well as Savonarola's children's army that spied on and denounced the population in the name of Christ. Among other reasons, Savonarola failed because of his criticism of the clergy and the pope; he was publicly burned to death in Florence.
In 1512, the Medici regained power in Florence, but another expulsion followed. Finally, in 1537, under **Cosimo I** (1519–1574), the Republic of Florence was transformed into a monarchy (Duchy of Tuscany, later Grand Duchy).

3 The Florentine master arithmeticians, especially the Calandri family[6]

The Florentine master arithmeticians in the 14th to 16th centuries, known as *abacisti fiorentini*, are concentrated in certain families, such as the "**Del Maestro Luca**", **Calandri** and **Micceri**, who passed on their knowledge to sons, nephews etc., but were also connected with each other through marriage.
The first appearance of the Calandri family is documented in the 13th century with a certain Guadagno (p. 29), around 1321 a Florentine citizen named Calandro (p. 29) is mentioned, the family soon becomes wealthier, buys real estate, fields and vineyards. There are three outstanding master arithmeticians in the Calandri family: Calandro and his two sons Piermaria and Filippo Maria.
Not much is known about Calandro's father **Piero** (1393/96–1438): In 1415 (p. 30), he married the daughter Francesca (Checca) (ca. 1400–1470/80) of Maestro Luca

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5 Cf. Section 4.1. Giuliano was the at first intended dedication addressee of *Il principe* by Niccolò Machiavelli that was finally dedicated to Giuliano's nephew Lorenzo di Piero.

6 This section is arranged according to Ulivi 2013, p. 29–56, 93–106, 125–128 (family trees). The page numbers quoted refer to this book. Information without page numbers is taken from the family trees.

(1356–1433/37), had four children who reached adulthood (p. 31) (Calandro, Antonio, Lucia/Cecilia and the later monk Agnolo/Angelo (p. 34, 42)), took part in municipal committees and was honorably nominated for election. His exact profession is not documented, he fell into debt and died early. His widow Checca and his younger son Antonio – supported by Calandro himself – successfully continued Piero's business.

Calandro (1419–1468-03-18) and Antonio (1422/25 – ca. 1480) married the sisters Lucrezia (ca. 1430–1494) and Margherita (ca. 1432–1469/80) from the wealthy notary family of Agnolo/Angelo da Terranova[7] in 1448 and ca. 1450 respectively (p. 32). These connections contributed significantly to the prosperity of the Calandri brothers. They lived with their families and their mother Checca until at least 1458 in the inherited palazzo in Via Pietrapiana (still called that today; parish of San Pier Maggiore, p. 39) where Calandro remained until his death while Checca, Antonio and his family moved to Antella (comune di Bagno a Ripoli) in the nearby countryside (p. 33, 39).

In his function as a merchant, Calandro was admitted to the *Arte della Lana* (wool guild) in 1453 together with Antonio (p. 33). Calandro's main profession was *maestro d'abaco*, trained by his grandfather Maestro Luca and Luca's son Maestro Giovanni (1395–1437) (p. 35). Calandro was praised for his teaching skills; in addition to school, he was probably also active as a *misuratore* (land surveyor, p. 36). He taught in three school sites, initially in two rented ones and later in one of his own.

Calandro's first place of work, the *Scuola del Lungarno* (p. 35) (today Palazzo Gianfigliazzi Bonaparte, Lungarno Corsini 4–6, between Ponte Santa Trinità and Ponte Carraia) was a training center of great renown. Maestro Luca and his son Giovanni had already held school there (p. 94). From 1438 on, Calandro supported his uncle Giovanni (p. 93). Around 1440, he taught the eleven-year-old Benedetto da Firenze (1429–1479) (p. 94). In 1441, there was a contract between Maestro Mariano di Michele, Antonio Micceri and Calandro (p. 93). After several changes of ownership, the building belonged to the Spini in 1442 (p. 95). In 1447, the school was closed (p. 96).

From 1448 to 1459/63, Calandro rented a new site, the *Scuola di Piazza dei Pilli/Pigli* (p. 35, 96/97) (area destroyed at the end of the 17th century; was located where today Via Pellicceria, via Calimala and Mercato Nuovo (Piazza della Repubblica) meet (p. 99)); it is not clear whether the rental contract was interrupted.

Then, in 1459/63, Calandro bought a house in the immediate neighbourhood of the previous school and founded his own school there, later called the *Scuola della Corticina* ('alley') *d'abaco* (p. 98) that he ran until the end of his life (p. 35). Niccolò Machiavelli was taught there in 1480/81 (p. 101). Ulivi assumes that, after Calandro's death in 1468, Benedetto da Firenze taught here first (p. 35, 101), before Calandro's sons began to teach themselves.

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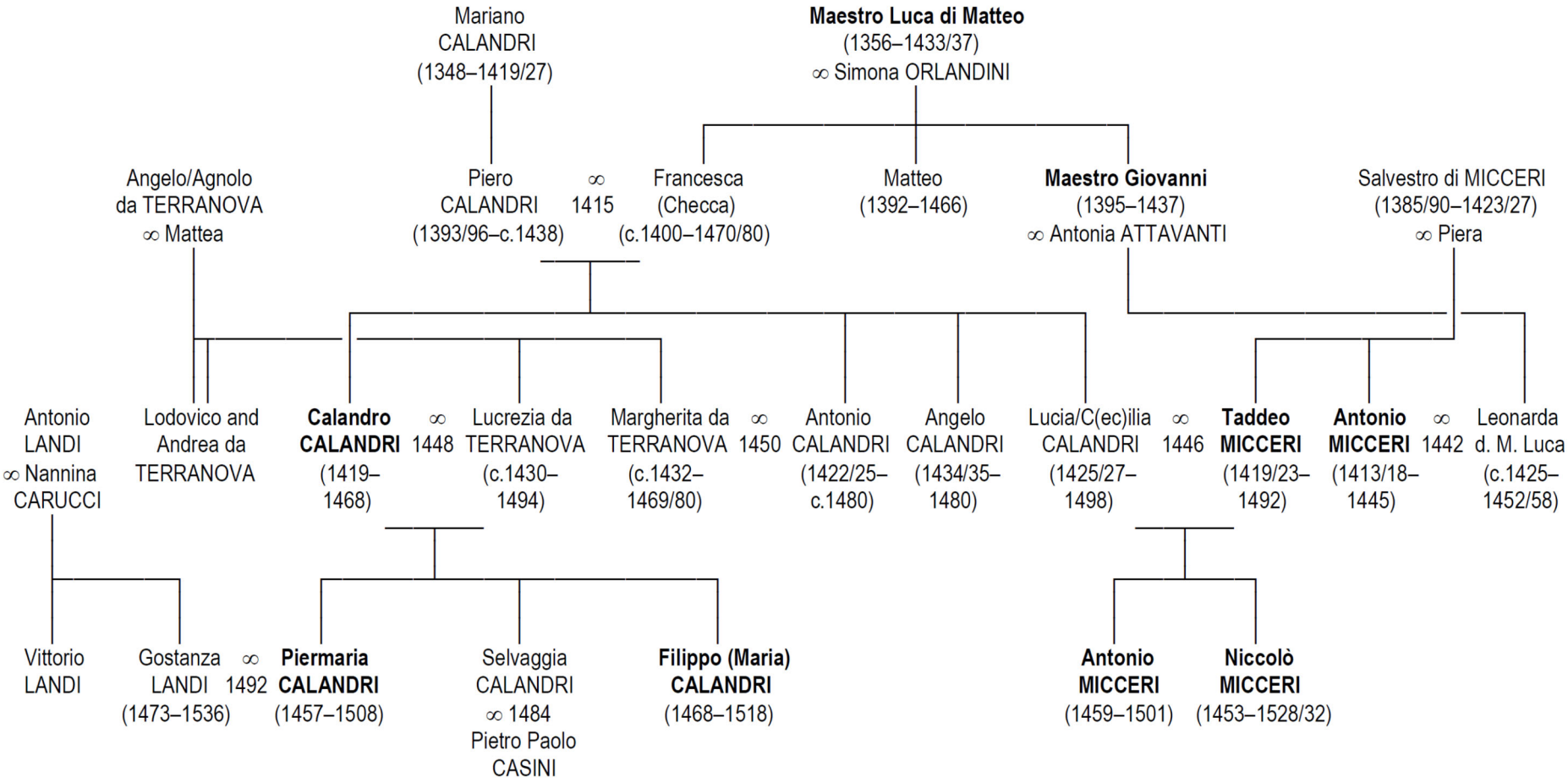
7 Lucrezia's and Margherita's brothers Lodovico and Andrea themselves were notaries.



The Calandri family and its relations to the Maestro Luca and Micceri families (the *maestri d'abaco* are highlighted in bold type)

Three children of Calandro and Lucrezia reached adulthood (Piermaria, Filippo Maria and Selvaggia).
The sons **Piermaria** (Florence 1457-03-06 – 1508-02-02 Florence) and **Filippo Maria** (Florence 1468-01-07 – 1518-04-13 Rome), both born in Via Pietrapiana (p. 40), were probably trained as *abacisti* by their uncle by marriage Taddeo dei Micceri (1419/23–1492)[8] after the early death of their father Calandro (p. 41). Piermaria began teaching in the school at the *Corticina dell'abaco* in 1480 (p. 41), and so did Filippo in the mid-1480s.
The sons took over the school of their father Calandro in 1487 (p. 41). They became very prosperous, either through inheritance or through Piermaria's marriage (1492/93) to Gostanza (1473–1536) from the wealthy Landi family.[9] Piermaria's marriage produced seven children (p. 44). Filippo Maria probably remained single (p. 45). The brothers took on further tasks, professionally as *calculatore* (person carrying out calculations) and *ragioniere* (accountant), politically as *sindaco* (mayor) and *gonfaloniere* (chairman of a commission), ecclesiastically as *elemosiniere* (administrator of donations) (p. 49). Piermaria and Filippo Maria remained in close contact throughout their lives, living in the same house in Via Pietrapiana (p. 51). In 1508, with Piermaria's death, the joint school ended and the building was sold (p. 44). Filippo took on other tasks (p. 103).


Coat of arms of the Calandri family

4 The works of the two Calandri brothers[10]

Nothing is known about mathematical manuscripts by Calandro di Piero, the father of the two brothers. There is only a large manuscript[11] of 500 leaves compiled by one of his students, Benedetto da Firenze (1429–1479), that can give an impression of the subjects taught in Calandro's abacus school.
Regarding the works of the brothers, there were probably many mutual influences as they lived and worked together in the same house until Piermaria died in 1508.

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8 A brother of Francesca (Checca), Maestro Giovanni, had a daughter Leonarda who married the master arithmetician Antonio, a son of Salvestro di Micceri, in 1442. Taddeo, an elder son of Salvestro, married Lucia (or Cecilia), a sister of Calandro.

9 Gostanza was the daughter of Antonio Landi, married to Nannina Carucci. Nannina's brother Luca was a notary (p. 43) so that the Calandri family was once again connected with a notary family (as with the da Terranova family).

10 Bibliographical details and editions can be found in the references. Remark regarding the transcriptions in Section 4: The Italian mathematician and historian of mathematics Gino Arrighi (1906–2001) describes and motivates his transcription principles that we adopt for this paper: He keeps the individual letters, but uses today's accents, apostrophes, upper and lower cases, word spaces and punctuation in favor of a better and more comfortable readability (Arrighi 1976, p. 11). In addition to Arrighi, we distinguish between *u* and *v* in this paper.

11 *Trattato* (Siena, Biblioteca Comunale degli Intronati, Cod. L. IV. 21), only small parts edited, cf. Egmond 1980, p. 189–190.

4.1 Filippo: *De arimethrica opusculum*, Florence 1492

This textbook is dedicated to Giuliano de' Medici (Florence 1479–1516 Fiesole) – at the age of 13 years –, son of Lorenzo il Magnifico and Clarice Orsini.[12]

The year number in the date of finishing the print (1491-01-01) indicated in the colophon[13] has to be interpreted with the so called *Calculus Florentinus*: The year number did not change on the 1st of January, but on the beginning of the new year scheduled on the 25th of March (Feast of the Annunciaton); that is, in the period between 1st of January and 24th of March, the year number of the year before was used.[14] This information can even be found in *De arimethrica* itself: *L'anno a Firenze si muta per la donna di marzo, cioè a di 25 di marzo* (pdf 116). That is, 1491-01-01 in Florentine style is 1492-01-01 in standard style. The 2nd edition 1518 appeared shortly after Filippo's death.

Filippo was only 23 years old when his textbook was printed; *in questa mia giovenile età* (juvenile age), he writes in the dedication.

De arimethrica treats typical problems of business and recreational mathematics of this period (see the following overview) – illustrated with woodcuts – and, insofar, it does not contain any mathematical surprise; with one exception: The rich geometry part is something new at this time.[15] And therefore, we examine it in detail (see the following comparison table). It shows many similarities with the geometry part in the codex Ricc. 2669 (see 4.2): fifteen problems coincide exactly and fourteen only differ in the number values – whereas the sequence of the problems is not the same.[16] These similarities induce two assumptions, namely that Ricc. 2669 was used as a source for *De arimethrica* and that Ricc. 2669 – which does not indicate any author – is written by Filippo. In the other parts, the similarities are not so striking.[17] They could also have their origin in a pool of more or less common mathematical problems. On the whole, *De arimethrica* is richer in mathememathical problems than the codex. E.g. Ricc. 2669 does not contain any correspondences to the sections [1], [2], [8] and [14] of *De arimethrica* (see the content overview).

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12 Cf. Section 2.

13 Inscription at the end of a book that can contain facts relative to its author, title, printer, publisher, place of publication, year or date of publication.

14 Grotefend 1960, p. 13.

15 An earlier geometry part in a printed arithmetic appears only in Johann Widmann, *Behend und hübsch Rechnung* (German), Leipzig 1489 (ca. 60 p.); a little later, we find geometry parts in Francés Pellos, *Compendion de lo abaco* (Occitan), Torino 1492 (16 pages with 39 examples), and Luca Pacioli *Summa de arithmetica, geometria, proportioni e proportionalità* (Italian), Venice 1494 (ca. 150 p.).

16 Piermaria's codex Acq. e doni 154 (see 4.5) also contains a geometry part, but the coincidences with *De arimethrica* are not that conspicuous as in Ricc. 2669. It would be interesting to check the geometry part of Ashb. 359 (see 4.6) as well, but there is neither a digital copy nor a complete edition.

17 Arrighi only gives the very vague argument that *De arimethrica* and Ricc. 2669 coincide in the form of the illustrations and the distribution of certain parts of the contents (Arrighi 1969, p. XVI f.).

Philippi Calandri ad nobilem et ſtudioſuz Julia
num Laurentii Medicē de arimethrica opuſculū.

¶ Philippi Calandri ad nobilem & ſtudioſum Iulianū
Laurentii Medicem de Arimethrica opuſculum.

(2nd edition Florence: Bernardo Zucchetta 1518-07-20)

Impreſſo nella excelſa cipta di Firenze per ſ
Lorenzo de Morgiani et Biouanni
Thedeſco da Maganza fi
nito a di primo di
Gēnaio 1491

Filippo Calandri: *De arimetrica opusculum*. 1st ed. 1491/1492-01-01, title page, colophon
Pictagoras arithmetrice introductor
Philippi Calandri ad nobilem et studiosum[18] *Julia-*
num Laurentii Medicem de arimethrica opusculum.
Impresso nella excelsa cipta di Firenze per Ser
Lorenzo de Morgiani et Giovanni
Thedesco [Johann Petri] *da Maganza* [*Magonza*, Mainz] *fi-*
nito a di primo di
Gennaio 1491

[18] In the word *studiosum*, the letter m is vertical (ʒ) in the 1st ed. and horizontal (*m*) in the 2nd ed.

Content overview of *De arimethrica opusculum*

Comparison of the geometry parts in *De arimethrica*, Ricc. 2669 and Acq. e doni 154
(black shaded: equal problems; grey shaded: similar problems)

Subject	***De arimethrica***	**Ricc. 2669**	**Acq. e doni 154**
Basic terminology of geometry	--	LXI–LXII	208^{r}–212^{r}
Equilateral triangle (*triangolo equilatero*), area, altitude (*perpendiculare*)	pdf 191	LXIII	212^{v}
Isosceles (*equic*[*r*]*urio*) triangle, area	pdf 191	LXIIII, LXV	213^{r}
Irregular triangle (*scudo*) 13 14 15, area	pdf 192	LXVI	213^{v}
Irregular triangle, area, other way	--	LXVII	214^{v}
Rectangular triangle, hypotenuse	--	LXVIII	--
Rectangular/irregular triangle, incircle	pdf 206	LXVIIII	--
Circle, inscribed square	pdf 207	--	--
Triangle, inscribed square	pdf 207	LXX	--
Equilateral triangle, center	--	LXXI	--
Sector of a circle, wedge (*gherone*)	--	LXXII	--
Equilateral triangle, incircle	--	LXXII	--
Square (*quadro*), diagonal	pdf 190	LXXIII	--
Rectangle (*quadrilatero*), diagonal	pdf 190	LXXIIII	--
Circumcircle (*tondo*) of a triangle	--	LXXIIII	--
Arc (*arco da una ruota*)	pdf 194	LXXV	222^{v}
Equilateral triangle, circumcircle	pdf 206	LXXV	--
Circle (*tondo*), area, diameter given	pdf 192	LXXVI	216^{r}
Circle, area, circumference given	pdf 193	LXXVI	216^{r}
Sphere (*palla*)	pdf 202	LXXVII	219^{v}
Cuboid (*pietra quadra*)	--	LXXVIII	--
Cone (*piramide tonda*)	pdf 199	LXXVIII	218^{v}
Round pillar (*colonna tonda*)	pdf 202	LXXVIIII	218^{r}
Wall of bricks (*muro di mattoni*)	pdf 197	LXXVIIII	--
Digging a well, costs	pdf 197, 139	--	118^{r}
Tiling a floor (*amattonare*)	pdf 194	LXXX	109^{r}
Round well (*pozzo tondo*)	pdf 200	p. 162	217^{r}
Cuboid (*canale*)	pdf 201	p. 163	217^{v}
Tub (*tino*)	pdf 201	p. 165	217^{v}
Cuboid/box (*cassa*)	pdf 198	p. 167	216^{v}
Barrel (*botte*)	pdf 200	p. 168	218^{r}
Sack (*sacco*)	pdf 198	p. 170	--
Rope from tower (*torre*) across river (*fiume*)	pdf 204	p. 171	221^{r}
Two barrels (*due botti*)	pdf 205	p. 172	223^{r}
Cutting a tree	pdf 205	--	--
Tent/pavilion (*padiglione*)	pdf 208	p. 173	--
Sphere, inscribed tetrahedron (*palla, piramide equilatera triangulare*)	pdf 208	--	--
Ladder at a wall	--	--	222^{r}
Broken tree (*albero rotto*)	pdf 204	p. 175	222^{v}
Cone (hill of crops – *monte di grano*)	pdf 199	p. 177	219^{r}
Two towers (*due torri*)	pdf 209	p. 178	--
Piece of land (*campo, pezzo di terra*)	pdf 195	p. 180	--
Coat (*mantello*)	pdf 195	--	223^{r}
Cube (*cubo*)	pdf 196	--	--
Cellar vault (*volta*)	pdf 196	--	--
Sphere in a cuboid (*palla, vivaio*)	pdf 203	--	--
Cubes in water (*pozzo, vivaio*)	--	--	220^{r} [2 probl.]
Two sacks (*due sacchi*)	pdf 203	p. 181	220^{v}
Measuring at a distance	--	--	223^{v}, 224^{v}

4.2 Filippo (probably autograph): *Aritmetica*, Florence ca. 1488 (Firenze, Biblioteca Riccardiana, Codice 2669)

The author is not indicated in this beautiful and exuberantly illuminated codex. The argumentation in 4.1 regarding the geometry part, however, combined with the strong relationship to the House of Medici in *De arimethrica* and the codex leads to the assumption that it was written by Filippo.[19] *De arimethrica* is explicitly dedicated to a Medici and the codex contains numerous references to the House of Medici: the coat of arms since 1465 (or [gold], five balls in orle [red] gules [forming a bordure], in chief [at the top] a larger one of the arms of France (azure, three fleurs-de-lis or)), the cut and blooming green branch, diamond rings, mottos, the skyline of Florence with the dome of the cathedral etc. (the latter two not represented in the figures in this paper). The emblems are either illuminated or sketched. There is a third reason to assume a Calandri as the author: Calculations are presented with red ink in irregular colored frames. This seems to be a tradition of the Calandri brothers as it can equally be found in the codex Acq. e doni 154 (see 4.5) that is explicitly assigned to Piermaria.

Filippo Calandri: *Aritmetica*. Florence ca. 1488. First page and squares (with emblems of the House of Medici: the coat of arms in the middle and the cut and blooming green branch) (Firenze, Biblioteca Riccardiana, Cod. 2669, i^{v}, vi^{v})

[19] Cf. the argumentation in Arrighi 1969, p. X–XII, XVI.

Filippo Calandri: *Aritmetica*. Florence ca. 1488. Incipit:
Multiprica 57 vie 7/8, fa'
così. Multiprica 7, che è
sopra alla linea de'
7/8, vie 57; fa 399 et
questo parti per 8, ch'è sotto detta linea, che
ne viene 49 7/8 e tanto fa.
[transcription according to Arrighi 1969, p. 3]
Calculation in a frame formed of a cut and blooming golden branch. At the bottom the coat of arms of the House of Medici with diamond rings, cut and blooming green branch.
(Firenze, Biblioteca Riccardiana, Cod. 2669, 1r)

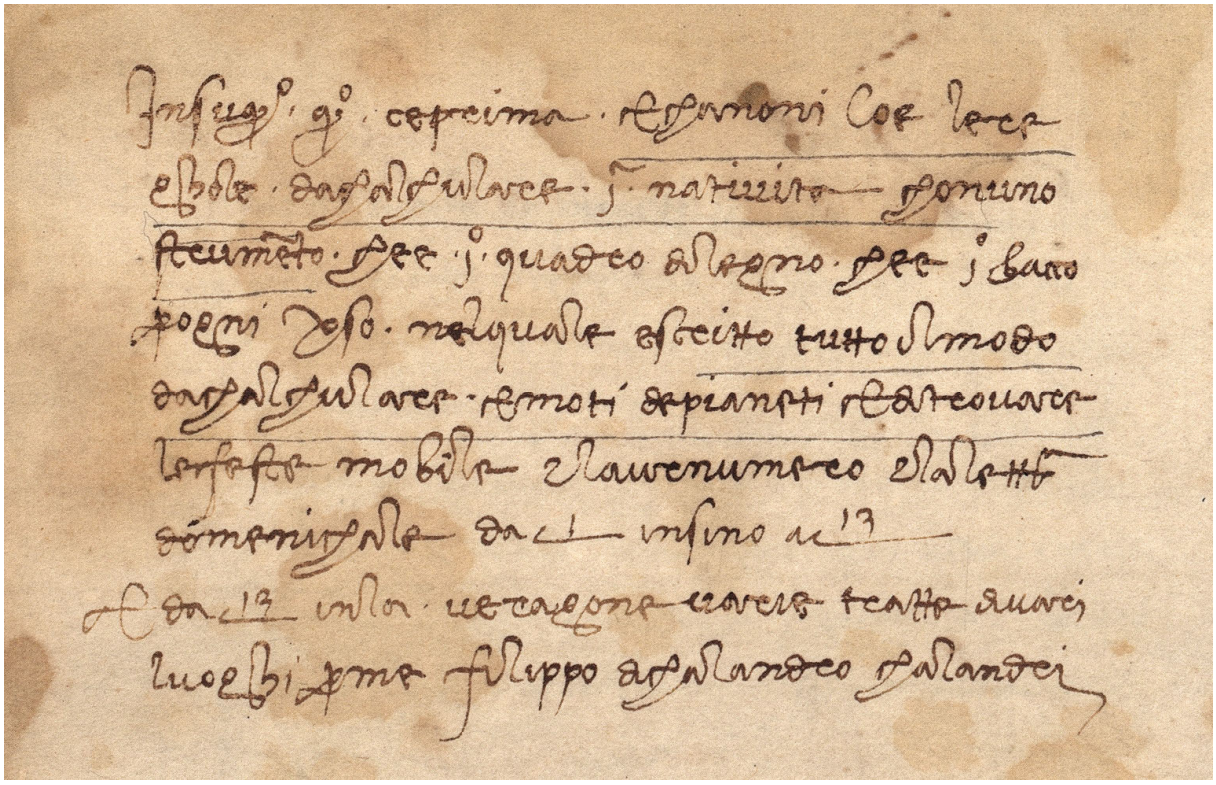
Comparison between Ricc. 2669 and Acq. e doni 154:
Beginning of the problem of the encounter of two ships and *La scuola d'abaco*
(the teacher holds a flagrum-like instrument of punishment in his right hand).
The frames around the calculations written with red ink are clearly visible.
On the left, sketches of the coat of arms of the Medici, perhaps the tower of the Palazzo Vecchio.
Left: Filippo Calandri: *Aritmetica* (Ricc. 2669, 89^{v}, 99^{v})
Right: Piermaria Calandri: *Tractato d'abbacho* (Acq. e doni 154, 87^{r}, 107^{v})

4.3 Filippo (autograph): *Raccolta di ragioni*, Florence ca. 1495 (Siena, Biblioteca Comunale degli Intronati, Codice L. VI. 45)

The codex consists of 112 unnumbered leaves and comprises three parts:

1 An anonymous cosmological treatise (1[r]–60[v]) that differs a lot in subjects and hand from the following two autograph parts by Filippo Calandri.

2 An astronomical treatise for the calendar, *Chanoni*, (61[v]–74[r]) that presents calculations for movements of planets, the identification of moveable holidays, that is Easter, of the golden number[20] and of the dominical letter[21].



Filippo Calandri: *Raccolta di ragioni*. Florence ca. 1495.
Incipit of *Chanoni e ragone*:
In su questo questo [sic!] *c'è prima e* [sc. *i*] *chanoni coè le re-*
ghole da chalchulare una natività chon uno
strumento che è uno quadro di legno che è uno bracco
per ogni verso nel quale è scritto tutto il modo
da chalchulare e [sc. *i*] *moti de' pianeti e di trovare*
le feste mobile e l'aur numero e la lettera
domenichale da c. [sc. *carta*] *1 insino a c. 13*
e da c. 13 in là v'è ragone varie tratte da vari
luoghi per me Filippo di Chalandro Chalandri
[transcription according to Santini 1982, p. II]
(Siena, Biblioteca Comunale degli Intronati, Cod. L. VI. 45, 61[v])

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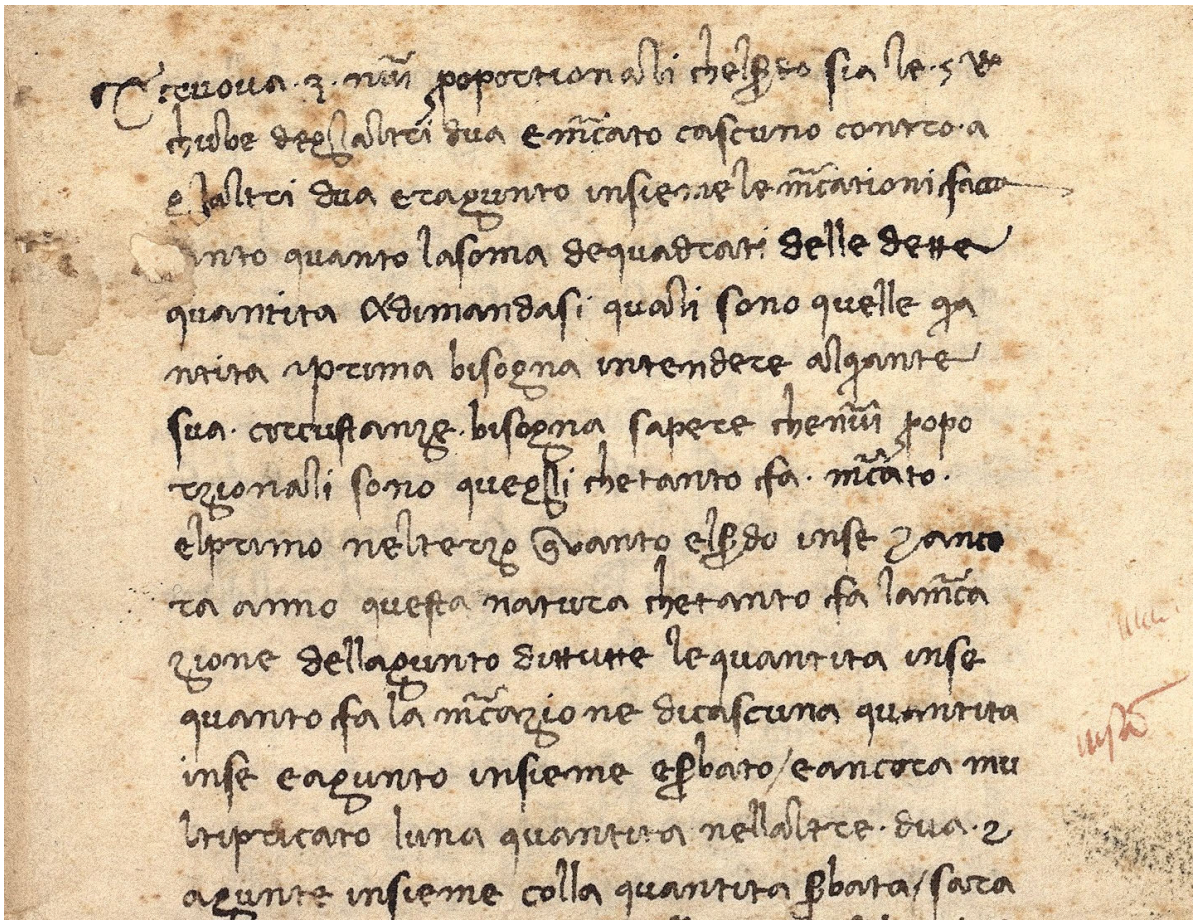
[20] Indicator of a year in a lunar calendar (necessary for identifying the Easter date) according to the 19 year Metonic cycle: The golden number (range 1 to 19) – calculated (year number + 1) mod 19 – is used to identify the moon age (epact, number of days after new moon) on January 1st and hence the dates of all calendric new moons. Golden number 1 means that calendric new moon is on January 1st.

[21] Indicator of a year in an eternal calendar. The dominical letter (range A to G) is the alphabet letter of the January Sundays in a year when January 1st gets the day letter A.

3 A mathematical treatise, called *Ragone varie tratte da vari luoghi per me Filippo di Chalandro Chalandri* ('various rules taken from various sources by me Filippo Calandri, son of Calandro') on 61^{v}, (75^{r}–111^{v}) that contains 71 unnumbered advanced problems often solved with algebraic methods. This collection of problems is nowadays entitled "Una Raccolta di Ragioni" ('a collection of rules').



Filippo Calandri: *Raccolta di ragioni*. Florence ca. 1495.
Incipit of *Ragone varie*:
[1] *Truova .3. numeri proportionali ch'el secondo sia le .5. R*
chube degl'altri dua e multipricato cascuno contro a
gl'altri dua e ragunto insieme le multipricationi facca
tanto quanto la soma de' quadrati delle dette
quantità. Adimandasi quali sono quelle qua-
ntità e prima bisogna intendere alquante
sua . circustanze . bisogna sapere che numeri propo-
rzionali sono quegli che tanto fa . multipricato .
el primo nel terzo Quanto el secondo in se' et anco-
ra ànno questa natura che tanto fa la multiprica-
zione dell'agunto di ttutte le quantità in se'
quanto fa multipricazione di cascuna quantità
in se' e agunto insieme e serbato / e ancora mu-
ltipricato l'una quantità nell'altre .dua. et
agunte insieme colla quantità serbata / sarà […]
[transcription according to Santini 1982, p. 1]
(Siena, Biblioteca Comunale degli Intronati, Cod. L. VI. 45, 75^{r})
[Comment on the problem in Santini 1981, p. 57–60, Santini 1982, p. VI–VII]

There is no digital copy of the entire manuscript as well as no edition of the first and the second part. Only the third part (*Ragioni*) was edited by Daniela Santini in her "tesi di laurea" (PhD thesis) in 1981 and in the officially published condensed version of her thesis – without the "note" (Santini's 'comments') – by Servizio Editoriale dell'Università di Siena in 1982.

Santini numbered the problems in her two editions, unfortunately in an inconsistent way, in 1981 from (1) to (70), in 1982 from (1) to (71), so it is not comfortable to use both editions in parallel. This is due to the fact that the problem on 85^{v}, which is crossed out in the original, was not counted in 1981 and was given the number (21) in 1982, thus shifting all higher numbers by 1. In the following, we use the numbering from (1) to (71) from Santini's second edition.

The order of the problems in the *Ragioni* is random; they are not arranged by subject or, for example, by level of difficulty. There are no structuring headings, apart from the indications *chasi insolubili* or *insolubile* from 105^{r} on (see below). Most of the problems do not appear in other works by Filippo. According to Egmond 1980, p. 193, and Santini 1981, nine pages carry geometrical diagrams.

The solutions presented by Filippo are, as already Santini noted, "sibylline", which is probably a polite way of saying "unmotivated" and "unfounded". Formulas that were well known at that time (e.g. for equilateral triangles) are neither quoted nor derived, but simply used. The calculations are hardly motivated; the problems, which sometimes extend over several pages, are not structured. The fact that problems and solutions are not formulated in a didactic way suggests that Filippo himself compiled this collection for his own use, for example to improve them later on or to direct them to more advanced students.

The *Ragioni* deal with mathematical problems from algebra and geometry, often focusing on commercial questions or relationships in equilateral triangles. In order to get a more precise overview of the problems, they have to be categorized. The classifications in Santini 1981 and Santini 1982 differ in part and are incomplete; therefore, we present a completed and partly refined categorization on this basis:

Trovare uno o più numeri soddisfacenti a certe proprietà (Find one or more numbers that possess certain properties): 1 (proportion), 18, 19 (proportion), 33, 34, 37 (wages, proportion), 39, 52 (proportion), 54 (cubic roots of 7 and 5), 55 (square numbers), 57 (square roots of 7 and 5), 58 (square number), 59 (sum of two square roots is 6), 60 (additive decomposition of 10, proportion)

Geometria (geometry): 2 (incircle), 3 (circumcircle), 9 (circumsphere), 10 (similar triangles), 11 (Pythagoras), 20, 21 (truncated tree), 22 (circle), 28 (fountain between two towers), 40 (incircle), 41 (scalene triangle), 53 (nonagon), 62 (circle with five incircles)

Compagnie (companies, proportional distribution): 4–7, 27, 29, 30, 56

Baratto (barter): 8, 42, 45–48, 61

Probabilità (division of stakes in gambling): 12, 43

Cambio (currency exchange): 13 (regula equalitatis), 70

Interesse (interest): 15, 49 (compound interest), 50 (compound interest), 63-69, 71

Nesting problems (first from last):
- Viaggi (travel): 16, 35
- Testament (1000*n*+1/10): 26

Questioni di denaro (questions about money):
- The amount of money that different people possess: 23 (give and take), 24 (give and take), 25 (One alone cannot buy: joint purchase – horse), 36 (relationship between two amounts), 44 (give and take)
- Distributing money 31, 32

Buying and selling apples: 14 (two sales), 17
Executing a testament: 51
Digging a well: 38

On 105^{r}, 14 problems labeled by Filippo as *chasi insolubili* ('unsolvable cases') begin, namely problems (49) to (62). *Chasi insolubili* is the heading before problem (49); for problems (50), (51) and (53) to (56), Filippo again explicitly notes the attribute *insolubile* as a heading and/or as a comment at the end; he classifies problem (52) as *inpossibile* at the end. For problems (55) and (57) to (62), there is no solution at all. Filippo explicitly writes in (49) and (50) that up to that time no one had found a solution method. Some of the problems, however, were falsely considered unsolvable as they had already been successfully solved by contemporaries of Filippo (see some comments in the foreword by Santini 1982). In problem (52), Filippo justifies his judgment of insolvability with a contradiction derived from the problem definition.
Which mathematical criterion Filippo uses to classify a problem as *insolubile* is not clear from the *Ragioni*. From the irrational solution of problem (1), which Filippo solves and therefore does not classify as *insolubile*, it follows at least that "irrationality" of the solution is not a criterion for *insolubile*, as Santini 1982, p. V, assumes. A more precise understanding of the term *insolubile* (in Filippo's sense) would require a detailed study of these 14 problems, which must be reserved for a separate article or a new edition.

Santini's 1981 edition is a work that deserves recognition as it provides quick access to the *Ragioni*. In addition to the valuable diplomatic transcription of the problems and solutions from the manuscript, however, it only contains very brief mathematical commentaries. Additions and improvements are desirable in three regards.

1. In addition to Santini's literal transcription, an extra formulation in today's standard would be desirable for a better understanding of the problems and solutions – especially for people who are not familiar with the language and mathematical notation in Florence around 1500:

- Use of modern orthography (e.g. "caso" instead of *chaso*, "dentro" instead of *drento*, "di tutte" instead of *di ttutte*)
- Structuring of the problems, for example in question, considerations, solution, alternative solution, final result
- Resolution of abbreviations, such as *co* (cosa) for the unknown *x*, *ce* (censo) for x^2, *cu* (cubo) for x^3, *R* (radice) for square root

- Presentation of the problems in today's formal language
- Explanations of the units of measurement at Filippo's time (e.g. *braccio* 'arm length')
- Professional use of mathematical notation (e.g. numbering of equations, correct and complete indexing of the unknowns)

2. The names for problem categories should be chosen carefully. For example, the category "travel" is about profits/losses made while travelling, not about journeys themselves. An appropriate categorization should also be made for problems that are dressed up as everyday problems of applied mathematics although they are unrealistic and belong to recreational mathematics.

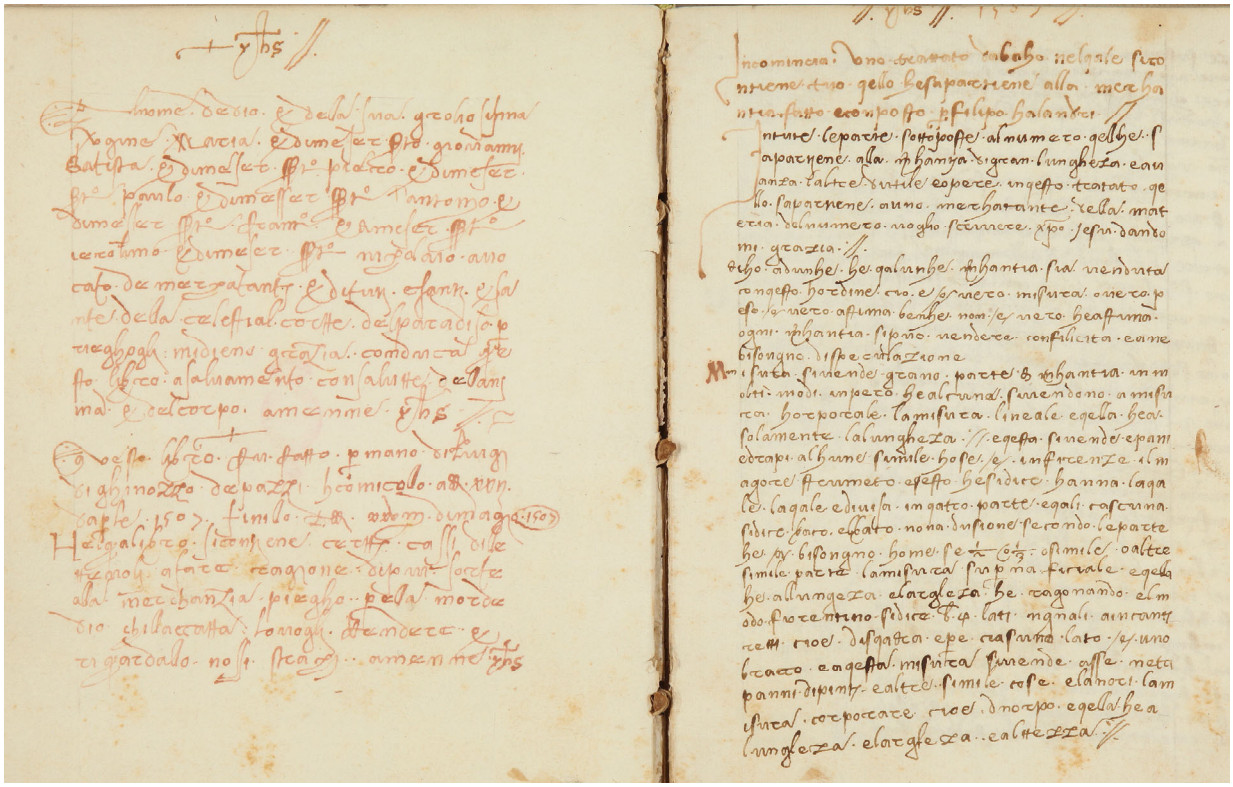
3. The solutions in the manuscript should be commented in such detail that they are comprehensible. In addition to these solutions, a contrastive and comparative presentation of further (obvious) solutions would be desirable. For example, according to Filippo, problem (37) in Santini 1981 (there with the number (36)) is solved in a quite complicated way and without motivated transformations. However, the problem could alternatively be solved very easily using a quadratic equation.

Even if these three reasons give rise to the desideratum of an expanded new edition of the *Ragioni*, Santini's transcription will nevertheless remain its basis.

4.4 Filippo (copied by Luigi di Ghinozzo de Pazzi): *Trattato d'abacho*, Florence 1507 (Firenze, Biblioteca Nazionale Centrale, Codice Magliabechiano XI.82)

The following three codices Magl. XI.82 (this section), Acq. e Doni 154 (see 4.5) and Ashb. 359 (see 4.6) have a similar explicit chapter structure (see the comparison table in 4.6) and nearly identical incipits.
Therefore, Egmond 1980 states that these codices as well as further 15 codices (the earliest Riccardiana 2109, ca. 1465, 11 *chapitoli*; Egmond 1980, p. 146) are not independent works, but largely based upon a not extant *Trattato d'abacho* by Benedetto da Firenze (1429–1479), a student of Filippo's and Piermaria's father Calandro (list of these codices in Egmond 1980, p. 356).

In 1507, some Luigi di Ghinozzo de Pazzi copied a not extant manuscript written by Filippo Calandri that is conserved as Magl. XI.82. As it comprises only 17 chapters, it seems to be incomplete compared to the other two codices with 23 chapters each. The chapter headings and the marginal calculations are written in read ink. The codex does not contain any illustrations. There is no edition.



Filippo Calandri: *Trattato d'abacho*. Copied by Luigi di Ghinozzo de Pazzi. Florence 1507.

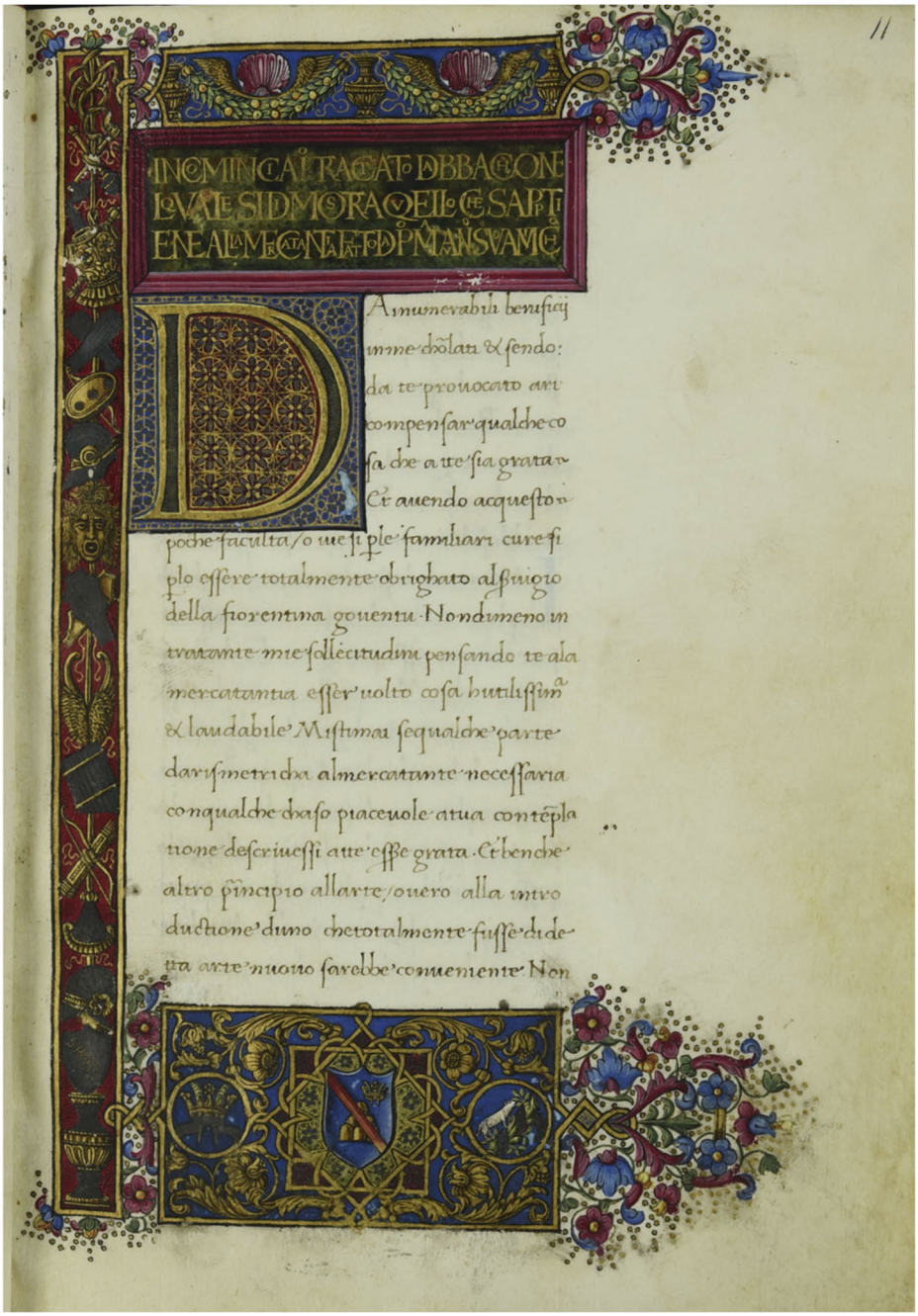
Inscription by Luigi di Ghinozzo de Pazzi:	Incipit: *yhs 1507*
Al nome de dio e della sua groliosis[*i*]*ma* [sic] *vergine Maria e di meser s*[*anc*]*to Giovanni Batista …* *Questo libro fu fatto per mano di Luigi di Ghinozzo de Pazzi, hcomicollo* [sic] *a ddi xxii d'aprile 1507, finillo a ddi xxviij di magio 1507. Hel qua libro si contiene certti cassi dillettevoli a fare ragione di più sorte alla merchanzia …* (iii^{v})	*Incomincia uno trattato d'aba*[*c*]*ho nel q*[*u*]*ale si contiene tu*[*t*]*o q*[*u*]*ello* [*c*]*he s'apartiene alla mer*[*c*]*hantia, fatto e conposto per Filipo* [*C*]*halandri.* *In tute le parte sotto poste al numero q*[*u*]*ell* [*c*]*he s'apartiene alla mer*[*c*]*hantia di gran lungheza e avanza l'altre d'utile …* (1^{r})

[transcription according to Egmond 1980, p. 111]
(Firenze, Biblioteca Nazionale Centrale, Cod. Magl. XI, 82, iii^{v}, 1^{r})

4.5 Piermaria (autograph): *Tractato d'abbacho*, Florence ca. 1482 (Firenze, Biblioteca Medicea Laurenziana, Codice Acquisti e doni 154)

The author's name is indicated on 11^{r} with the abbreviation *P^{o} M^{a}*. The dedication to some member of the family Ridolfi di Piazza, can be recognized from the coat of arms on 11^{r}. The manuscript contains illuminated multiplication tables, illuminated initials in the beginning of each of the 23 chapters, marginal calculations in red in colored frames, marginal geometrical diagrams in blue, ca. 40 pages with illuminated illustrations (see the figures in 4.2).

Arrighi 1976, p. 11, underlines the dedication, the big number of colored illustrations ("magnificamente illustrato") and the profounder presentation ("approfondimento della materia") and, therefore, considers this codex as a new edition ("rifacimento") of Ashb. 359 (see 4.6). Regarding the year of creation, the argumentation in Arrighi 1976, p. 10–11, is based on the seven years larger year numbers in this codex (192^{r}) compared to those in Ashb. 359 (177^{r}).



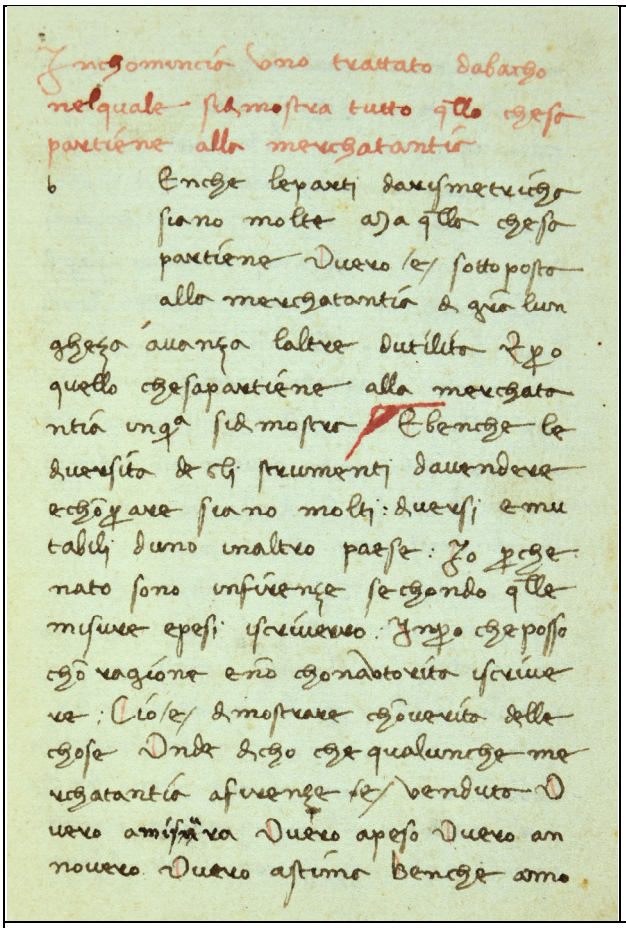
Piermaria Calandri: *Tractato d'abbacho*. Florence ca. 1482. Incipit:
Incomincia uno tractato d'abbacho ne-
l quale si dimostra quello che s'app[*ar*]*ti-*
ene alla mercatantia fatto da P^o M^a a uno suo amicho

Da inumerabili beneficij *in me chonlati et sendo* *da te provocato a ri-* *compensar qualche co-* *sa che a tte sia grata* *et avendo a cquesto* *poche facultà o vie sí per le familiari cure sí* *per lo essere totalmente obrighato al servigio* *della fiorentina goventù, nondimeno in-*	*tra tante mie sollecitudini, pensando te a la* *mercatantia esser volto, cosa hutilissima* *et laudabile mi stimai se qualche parte* *d'arismetricha al mercatante necessaria* *con qualche chaso piacevole a tua contempla-* *tione descrivessi a tte essere grata. Et benché* *altro principio all'arte o vero alla intro-* *ductione d'uno che totalmente fusse di de-* *tta arte nuovo sarebbe conveniente non*

[transcription according to Arrighi 1974, p. 31]
(Firenze, Biblioteca Medicea Laurenziana, Cod. Acq. e doni 154, 11^{r})

4.6 Piermaria (possibly autograph): *Trattato d'abacho*, Florence 1475 (Firenze, Biblioteca Medicea Laurenziana, Codice Ashburnhamiano 359)

The manuscript does not indicate Piermaria Calandri's name. It comprises 23 chapters. In contrast to Egmond 1980 (see 4.4), Arrighi 1976, p. 11, is convinced that this codex was written by the same author as Acq. e Doni 154 (see 4.5), that is, by Piermaria Calandri. Regrettably, there is no digital copy and no complete edition of this manuscript. Therefore, only the brief information from Arrighi's partial edition (1976) can be included in the comparison table with Acq. e doni 154 and Magl. XI.82 (at the end of this section). Especially, Arrighi 1976 presents hardly any details about the 30 pages geometry part that would have been important for the comparison with the geometry parts of *De arimethrica opusculum*, Ricc. 2669 and Acq. e doni 154 (see 4.1).



Inchomincia uno trattato d'abacho nel quale si dimostra tutto quello che s'apartiene alla merchatantia Benche le parti d'arismetricha siano molte, ma quella che s'apartiene o vero è sottoposta alla merchantantia di gran lungheça avança l'altre d'utilità; e però quello che s'apartiene alla merchatantia in questa si dimostra. E benchè le diversità degli strumenti da vendere e chonprare siano molti, diversi e mutabili d'uno in altro paese, io perchè nato sono in Firençe sechondo quelle misure e pesi iscriverrò. Inperochè posso chon ragione e non chon autorità iscrivere, cioè dimostrare chon verità delle chose. Onde dicho che qualunche merchatantia a Firençe è venduta o vero a misura o vero a peso o vero a nnovero o vero a stima, benchè anno
[transcription according to Arrighi 1976, p. 13]

Piermaria Calandri: *Trattato d'abacho*. Florence 1475. Incipit.
(Firenze, Biblioteca Medicea Laurenziana, Cod. Ashb. 359, 10[r])

Comparison of the chapters in Magl. XI.82, Acq. e doni 154 and Ashb. 359

Chapter	Contents (title from Acq. e doni 154)	Magl. XI.82	Acq. e doni 154	Ashb. 359
1	*Come pell'arti minori la mercatantia si vende* Minor artefacts (coins)	2^{v}	19^{v}	14^{r}
2	*Rotti* Fractions	5^{r}	25^{r}	21^{r}
3	*Proporzione di due quantità note*	11^{v}	35^{r}	31^{v}
4	*Ragioni apartenenti a rotti*	17^{v}	42^{r}	40^{r}
5	*Regola di 3 chose* Rule of three	22^{r}	50^{r}	47^{r}
6	*Casi dilettevoli di numeri*	27^{r}	55^{v}	52^{v}
7	*Soldi a oro* Golden coins	31^{v}	61^{r}	58^{v}
8	*Casi piacevoli absoluti colla posizione semplice* Simple false position method	48^{r}	82^{r}	80^{r}
9	*Come pell'arti maggiori la mercatantia si vende* Major artefacts (coins with precious metal content)	55^{r}	90^{v}	86^{v}
10	*Casi piacevoli absoluti per varie regole*	69^{v}	106^{v}	101^{v}
11	*Ridurre uno peso o una misura d'uno in un altro paese* Converting a weight or a measure	74^{r}	111^{v}	106^{v}
12	*Casi piacevoli per varie regole absoluti*	78^{r}	117^{r}	112^{r}
13	*Baratti* Barter	81^{r}	120^{v}	115^{v}
14	*Dividere il guadagno intra dua o più compagni* Companies	92^{r}	136^{r}	129^{r}
15	*Casi piacevoli absoluti per forza di numeri* [*indivinare* – Number guessing] [Reference to Leonardo pisano cap. 12]	--	146^{v}	138^{r}
16	*Consolare* [= *legare*] Metal alloys	98^{v}, Ch. 15	155^{r}	144^{v}
17	*Casi piacevoli d'uomini che hanno danari*	108^{v}, Ch. 16	167^{v}	157^{r}
18	*Meritare semplicemente e a capo d'anno* (*proibito secondo la legge cristiana*) Simple and compound interest (forbidden for Christians)	113^{v}, Ch. 17	175^{r}	163^{v}
19	*Scontare semplicemente e a capo d'anno* Simple and compound discount	--	182^{v}	169^{r}
20	*Casi piacevoli – uomini che hanno danari e vogliono comprare cavagli* One alone cannot buy: joint purchase – horse	--	187^{r}	172^{v}
21	*Recare a termine* Settlement on one day	--	191^{r}	176^{v}
22	*Casi piacevoli – uomini che hanno danari e truovono borse di danari* The found purse	--	204^{r}	188^{v}
23	*Geumetria*	--	208^{r}–225^{r}	192^{r}–207^{r}

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[Edition of 2 *Ragone varie* (complete)] Santini, Daniela: Filippo Calandri, *Una raccolta di ragioni*, dal Codice L. VI. 45 della Biblioteca Comunale di Siena (= Quaderni del Centro Studi della Matematica Medioevale 4). Siena: Servizio Editoriale dell'Università 1982, X, 50 p. (Siena: Tesi di laurea 1981, 116 p.).

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