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Alfred Holl

The earliest printed French, Dutch and English arithmetic textbooks

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The earliest printed French, Dutch and English arithmetic textbooks

Alfred Holl

1 Introduction

The goals of this paper are to briefly present the earliest printed French, Dutch and English arithmetics extant and to examine the main Continental sources of the earliest printed English arithmetics.

Two recent findings extend our knowledge about these issues:[1]

1 *L'art et science d'arithmétique*, Paris: Michel Toulouse ≤ 1496.[2] The book survived completely (112 p.), but in only one copy in public libraries. It is held by École Nationale Supérieure des Beaux Arts ENSBA, Paris.[3] Up until now, the two earliest generally known printed French arithmetics were *Livre de chiffres* 1501[4] and *Arismétique corrigée* 1512/15.

2 *Arte and science of arismetique*, London: Richard Faques 1526-03-13.[5] Only the last page with 1 $^{1}/_{3}$ lines of the last mathematical problem ("the three saints"), the complete colophon and the printer device survived. It is held by the British Library, London. Up until now, the two earliest generally known printed English arithmetics were *Introduccion* 1537[6] and its successor *Introduction* 1539.

In 1947, Allie Wilson Richeson (1895–1966; University of Maryland) already addressed the question of French sources of the earliest English arithmetics[7] (based on a discussion of *Introduction* 1539), but without mentioning any possible source. Paul Bockstaele (1920–2009; University of Leuwen/Louvain) verified in two famous papers (Dutch 1959, English 1960) that *Introduction* 1539 is the composition of the translations of a French and a Dutch arithmetic with a clearly recognizable cut between its two parts. He assumed *La vraye manière* ca. 1535 as the French source and *Die maniere* ca. 1510 as the Dutch source. Bockstaele did not know any earlier French arithmetic, nor *Arte and science* 1526, nor *Introduccion*

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[1] This paper has its origin in my research for Holl 2022.

[2] Cf. Holl 2022, p. 59–63. As far as I know, the book was not discussed anywhere else.

[3] For this and the following arithmetic books, see the bibliography table (5.1) for details. Place names are indicated in the local language and not in their English version, e.g. Antwerpen, not Antwerp, Lyon, not Lyons.

[4] Tropfke 1980, p. 699.

[5] Cf. Williams 2012, p. 165; Holl 2022, p. 154–156.

[6] Bought by the Friends of the British Library for 97,500 GBP in 2005.

[7] From now on, I use the word "arithmetic" instead of the expression "printed arithmetic (text-)book".

1537. Therefore, his assumption aimed at a French source that was printed too late than to have influenced the earliest English arithmetic (1526). Bockstaele's comparison, however, keeps its value as *La vraye manière* ca. 1535 has predecessors with similar contents. But following the temporal sequence of publications, we now see that the French (earliest arithmetic ≤ 1496) could have influenced the Dutch (earliest arithmetic 1508) and not the other way round as he might have thought – or: both used the same source.
In 2012, Travis D. Williams (University of Rhode Island) added a lot of information to Bockstaele's paper as he included *Arte and science* 1526 and *Introduccion* 1537 as well as the French arithmetics *Art et science* 1512/19 and *Art et science* ca. 1545. But Williams did not include the two earliest French arithmetics either and thus could not draw any correct conclusion on the relationship between French and Dutch arithmetics.
Against this background, my article is a revision of and an addition to the excellent research presented in Bockstaele's and Williams's papers. The kernel of my examination is a comparison of about ten arithmetics in French, Dutch and English in the form of the table in 4.2.
My comparison is structural – I aim at the architecture of the books – and not detailed – I do not consider philological details such as spelling conventions, wording, number values and units of currency and measure. The temporal focus is on the period until 1537 which is relevant for Continental influences on *Introduccion* 1537, the first English arithmetic which used both French and Dutch sources. I start with descriptions and comparisons of the earliest printed French, Dutch and English arithmetics.

2 The earliest printed French arithmetics

2.1 Description

The earliest printed French arithmetic extant is *Art et science* ≤ 1496.[8] Its title and its colophon run in English:

This is the art and science
of arithmetic with
examples and practice, very
useful and profitable for all
people and easy to
learn, with the subtle counter, recently
translated from Latin into French
and extracted from several volumes,
by means of which
everyone can teach himself and train
by himself

Here finishes the treatise
on the art and science of arithmetic with
examples and practice, easy to learn,
with the subtle counter, translated from
Latin into French, containing several
results as becomes apparent above,
recently printed in Paris by
Michel Toulouse dwelling
at Mont Saint Hilaire

The prologue starts with *Les philosophes disent que toutes choses qui sont ne jamais furent au monde ont este produites par nombre – The philosophers say that all things which exist were not in the world from the very beginning,* [but] *were*

[8] Cf. Holl 2022, p. 59–63. See the bibliography table (5.1) for the estimated the year of publication.

created by [the law of] *the number*.[9] Without the art of the number, we would not be able to understand them perfectly. This little treatise about the algorism would be very profitable to understand all sciences, especially geometry, astrology, music. It would be necessary for all people who have to calculate (*conter et gecter; faire contes et sommes*): treasurers (*trésoriers*), receivers [tax collectors] (*recepueurs*), citizens (*bourgoiz*), possibly illiterate (*ne scavent lire ny escripre*) merchants (*marchans*), changers (*changeurs*), negotiators (*négociateurs*).

Title of *Art et science* ≤ 1496

Counters and colophon of *Art et science*

After the prologue, the book is arranged in six numbered parts (*livres*):[10]

Part 1: Digits (*chiffres*) and six species (*espèces*)[11] with counters (*gects*) for integers (*nombres entiers*): numeration[12], addition (*adiouster*), reduction[13], subtraction (*sustraction*), multiplication, division

[9] The same prologue as in *Art et science* 1512/19, *La vraye manière* ca. 1535, *Art et science* ca. 1545.

[10] See the comparison table (4.2) for details.

[11] Basic arithmetical operations.

[12] Notation/representation of numbers: digits, number symbols, numerals.

[13] Ascending reduction (*reductio ascendens*): conversion of a numerical value/s given in lower units of a system of currency, measure, weight or time to a value/s in higher units (*Introduction* 1539, Evi^{v}: *to reduce the lesse* [*summe*] *to the more*). Descending reduction (*reductio descendens*): conversion from higher units to lower units (*Introduction* 1539, Ev^{r}: *to reduce the more summe to the lesse*).

Part 2: Six species (the same as in Part 1) with the pen (*par chiffres*; in later editions also *par la plume*) for integers; multiplication table (*petit livre des marchans*)
Part 3: Six species for fractions (*fractions, nombres rouptz*)
Part 4: Rule of three (*règle de trois*; 7 cases) for integers and fractions, measures (3 cases: long measures, measures of capacity (wine), loaf of penny (*pain d'un denier tornois*))[14] and weights
Part 5: Companies without time (*compaignies sans temps*; 1 case), companies with time (*compaignies à temps*; 3 cases), companies with subcontractors (*compaignies de facteurs*; 3 cases), bartering (*règle de changes pour éviter fraud, baratte*; 3 cases)
Part 6 (in later editions often falsely called *quatrième livre*): 34 numbered miscellaneous word problems (*règles*)

The book was printed in several editions during more than one century without any major modifications. Compared to *Art et science* ≤ 1496, the prologue was partly changed; regarding the calculation with the pen, the reduction for integers (amounts in a certain currency) was dropped; duplication, halving ("mediation"), arithmetic progression and a brief comment on root extraction were added; the number of cases for the rule of three was reduced from 7 to 4; the number and sequence of the miscellaneous word problems was slightly changed.[15]

Within the temporal focus of this paper, the following editions are known:[16]

Livre de chiffres 1501[17]
Art et science 1512/19
La vraye manière ca. 1535[18]
Art et science ca. 1545[19]

And beyond the temporal focus of this paper:

L'arithmétique et manière d'apprendre à chiffrer et compter par la plume et par les getz en nombre entier et rompu, Lyon: Thibault Payen 1548 [Hoock/Jeannin I/-5.1]
L'arithmétique et manière, Paris: Jean Ruelle (rue Sainct Jacques, à l'enseigne de la queue de regnard) 1551 [München BSB]
L'arithmétique et manière, Paris: Jean Ruelle 1556 [Jagellonian Digital Library; Hoock/Jeannin I/-5.2; possibly also 1555 (according to Smith, p. 268), 1559]
Anthoine Cathalan (maistre A. C. és Artz et Mathématiques professeur): *L'arithmétique et manière*, Lyon: Thibault Payan 1566 [online hist-math.fr; also 1561]
L'arithmétique et manière, Paris: Pierre Ménier 1585 [Hoock/Jeannin I/-5.4]
Anthoine Cathalan: *L'arithmétique et manière*, Lyon (en rue mercière, à l'enseigne de la sphère) 1599 [Praha Národní knihovna]

[14] The "loaf of penny", German "Pfennigbrot", (always costs one penny) is an example for the inverse rule of three: The more expensive a unit of corn is, the smaller this loaf is and the other way round.
[15] See the comparison table (4.2) for details.
[16] As far as editions can be found on the Internet. See the bibliography table (5.1) for details. The sequence of the editions is not correctly identified in Hoock/Jeannin I.
[17] This textbook was used as a reference by Tropfke 1980 so that all of the miscellaneous word problems occurring can easily be assigned to one of the categories in Tropfke 1980, Part 4. See the comparison table (4.2).
[18] The edition that Bockstaele (1959, 1960) thought to be a source of *Introduction* 1539.
[19] Williams 2012, p. 178–179, exhaustingly discusses the dating of the three latter editions, including genealogical facts of the printers' families.

Four other French arithmetics were published before *Introduccion* 1537. I do not discuss them as they have different structures as the mentioned row of editions and therefore belong to other traditions (shaded lines in the bibliography table (5.1)):

Arismétique corrigée 1512/15 (adopting the prologue of *Livre de chiffres* 1501)
Juan de Ortega, Claude Platin: *Oeuvre tressubtile* 1515 [based on Ortega: *Conpusicion* 1512]
Estienne de La Roche: *Arismétique* 1520.
Godefroyd Gomparst: *Un livre de compte* ca. 1535.

2.2 Multiplication and division of fractions in *Art et science* ≤ 1496

Art et science ≤ 1496 (pdf 25–27) contains a strange and erroneous peculiarity regarding the multiplication and division of fractions: The smallest common denominator is identified, and then only the numerators are multiplied or divided. That is, the method for adding and subtracting fractions is falsely literally transferred to multiplication and division. Examples:

$$2 \cdot \frac{3}{3} = \frac{6}{3} \cdot \frac{3}{3} = \frac{18}{3}$$

$$\frac{6}{4} : \frac{2}{3} = \frac{18}{12} : \frac{8}{12} = \frac{18/8}{12} = \frac{2}{12} + \frac{2/8}{12}$$

In the section on dividing fractions, there is an explicit reference to subtracting fractions. In the 1551 and 1556 editions, one even finds the subtraction idea instead of a division (sic!):

$$\frac{6}{3} : \frac{2}{3} = \frac{4}{3}$$

These mistakes are conserved throughout all of the following editions during more than 100 years. Neither Bockstaele 1959/60 (examining *La vraye manière* ca. 1535) nor Williams 2012 (examining *Art et science* 1512/19) recognized the mistakes. Claiming a check of the textbook by the professor of mathematics Anthoine Cathalan on the title page from latest 1566 on, had no influence on the mistakes. Perhaps Cathalan's name was only used for marketing, and he had never had a close look at the contents. Only one emendation was introduced from the 1556 edition on: The false multiplication above was replaced by:

$$9 \cdot \frac{2}{3} = \frac{18}{3}$$

The first French arithmetics outside the mentioned row of editions provide correct calculations for fractions (cf. 2.3).

2.3 Possible sources of *Art et science* ≤ 1496

The question for possible sources of *Art et science* ≤ 1496 arises especially due to its mistakes in multiplying and dividing fractions. Its title page claims that the arithmetic was *translatee de latin en francois et extraicte de plusieurs Volumes – translated from Latin into French and extracted from several volumes*. The reference to Latin sources could just be a promotion trick (like possibly Cathalan's name) in order to raise the reputation and thus the sales quantity of the book.

One possible source, however, can easily be found in form of the Latin textbook *De arte numerandi* ca. 1490.[20]
It comprises four numbered parts (*libri*) that remind of the first French arithmetic:[21]
Part 1: Six species with the pen for integers
Part 2: Six species for fractions – containing the same mistakes with multiplication and division of fractions (pdf 26–27)
Part 3: Five species with counters for integers
Part 4: 14 miscellaneous numbered word problems (including rule of three, loaf of penny, companies without and with time) all of which can be found in *Art et science* ≤ 1496 as well (ten of them in its Part 6) – besides problem 8 (ship purchase with proportional distribution) which shows up as problem 11 in the earliest Dutch versions.
On the whole, possible printed sources for certain word problems (quoted below with their numbers) in Part 6 of *Art et science* ≤ 1496 are (or: the French arithmetic and its possible source used the same common sources):
De arte numerandi. Paris ca. 1490: for 2, 4, 6, 8, 14, 16, 17, 21, 23, 24
Trento Algorism (in German). Ca. 1475: for 18, 24
Borghi, Piero: *Aritmetica*. Venezia 1484: for 2, 27
Calandri, Filippo: *Trattato*. Firenze 1491: for 4, 5, 16, 19 20, 21, 25, 27, 29
Pellos, Francés: *Compendion* (in Occitan). Torino 1492: for 15, 18, 22, 24, 27
Widmann, Johann: *Rechnung*. Leipzig 1489: for 2, 21
And a possible 15th century source that was not printed:
Algorismus Ratisbonensis: for 6, 15, 18, 19, 20, 21, 22, 24, 26, 27, 29

3 The earliest printed Dutch arithmetics

3.1 Description and comparison

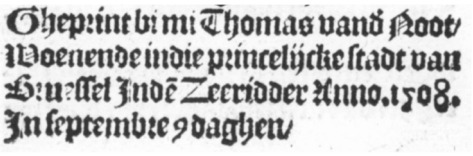
There is no new knowledge about the earliest Dutch arithmetics. The probably earliest one is *Die maniere* 1508.[22] Its title and its colophon run in English:

The way to learn to calculate[23] *according to the correct art of algorism.* *For whole and broken numbers*	*Printed by me Thomas van der Noot* *dwelling in the princely town of* *Brussel in the Zeeridder*[24] *in the year 1508* *on the 9th day of September*

The prologue starts with *Want onder die seuen vrie consten gheen* [2nd ed. *gheene*] *en es* [2nd ed. *is*] *die alleen wesen* [2nd ed. *staen*] *mach dan Arithmetica – As among the seven liberal arts there is none that could stand alone except arithmetic*.[25]
After the prologue, the book is arranged in three not numbered parts:[26]

[20] See the bibliography table (5.1) for details.
[21] See the comparison table (4.2) for details.
[22] Cf. Holl 2022, p. 144–148.
[23] Dutch verb *cyffren*, cf. French verb *chiffrer*.
[24] Name of a house.
[25] The prologue is the same as in *Introduccion* 1537; commented in detail in Williams 2012, p. 171.
[26] See the comparison table (4.2) for details.

Part [1]: Seven species (*bewerkingen*) for integers (*gheheele*) with the pen (*metter pennen*): numeration, addition, subtraction, halving ("mediation"), duplication, multiplication and division (with check by 7 or by inverse operation)
Part [2]: Seven species (the same as in Part 1, numbered 8 to 14) for fractions (*ghebroken ghetalen*)
Part [3.1]: [Basic rules] Rule of three (*ghulden reghel*, *reghel von tri*), companies without and with time (*reghel van gheselscape; reghel van den tide der ghesellen*), bartering (*reghel van barteringhen*)
Part [3.2]: [Other rules] 17 not numbered miscellaneous word problems



Title of *Die maniere* 1508

Colophon of *Die maniere* 1508

The 2nd edition, *Die maniere* ca. 1510, added Part [4] on five species (numeration, addition, subtraction, multiplication, division) with counters (*metten penningen*) and four problems on quantity-price relations of pins (*spellen*, today *spelden*), dice (*teerlingen*), saffron and alum in the end of the book.[27]
This edition was translated into French: *La manière* 1529.[28] The two extant copies held by the Bibliothèque Nationale Française and the Houghton Library at Harvard[29] are different prints of the same edition. Line organization, page organization, signatures and leaf numbers (*Gii*/*G2* in the end) are identical, the orthographical conventions differ.[30] A later Dutch edition was printed by Jan van Ghelen in Antwerpen in 1569.[31] The first Dutch arithmetic outside the mentioned row of editions is Gielis van den Hoocke, *Een sonderlinghe boeck* 1537 (shaded line in the bibliography table (5.1)).[32]

[27] Cf. Williams 2012, p. 183.
[28] All three of these books are discussed in Bockstaele 1959 and Bockstaele 1960.
[29] Bockstaele knew only this copy.
[30] Cf. Williams 2012, p. 177.
[31] Hoock/Jeannin I/-20.6.
[32] Cf. Wolfgang Kaunzner in Gebhardt, Rainer (ed.): Rechenbücher und mathematische Texte der frühen Neuzeit. Annaberg-Buchholz 1999, p. 103–114; Holl 2022, p. 149.

3.2 Possible sources of *Die maniere* 1508

So far known, two French arithmetics (*Art et science* ≤ 1496 and *Livre de chiffres* 1501) were printed before the first Dutch arithmetic.

Possible printed sources for certain word problems (quoted below with their numbers) in Part 3.2 of *Die maniere* 1508 are:

Art et science. Paris ≤ 1496: for [3], [4], [6], [9], [10] (shaded lines in the comparison table)
De arte numerandi. Paris ca. 1490: for [11]
Calandri, Filippo: *Trattato*. Firenze 1491: for [1], [2], [3], [4], [5], [6], [11], [12]
Widmann, Johann: *Rechnung*. Leipzig 1489: for [5], [6], [7], [8], [13], [14], [15]

This is no evidence that *Die maniere* 1508 copied some word problems from *Art et science* ≤ 1496. Both of the two arithmetics could also have used the same common sources.

4 The earliest printed English arithmetics

4.1 Description and comparison

The earliest printed English arithmetic extant is *Arte and science of Arismetique* 1526.[33] It survives as a single leaf that contains the colophon[34] with title, a reference to a French source (or several French sources), the printer (Richard Fakes, otherwise spelled Fawkes or Faques) and one of his devices, the place of publication (Durham Rent and St. Pauls Church yard in London), the finishing day of printing (13th March 1526) and the trailing 1 1/3 lines of the last word problem discussed ("the three saints"[35]). The text runs in today's English:

Thus ends the Art and science of Arithmetic
set out with goodly examples and rules
translated from French into English not without
great labour and study, to the intent that
merchants occupying beyond the sea should
have knowledge of their coins of money,
that is to say Crowns, Ducats and Saluts,
Francs, and with all other small
money after their value; and also
with the measures, both of corn and wine each
one after their measures as it shows [it] *more*
plainly in the said book. Imprinted
by me Richard Faques dwelling
in Durham Rent, or else in
Paul's church yard, at the sign of
the .A.B.C. and finished the year of
our lord God 1526, the
13th day of March.

[33] Discussed in Williams 2012, p. 165–166, with genealogical facts of the printer's family.

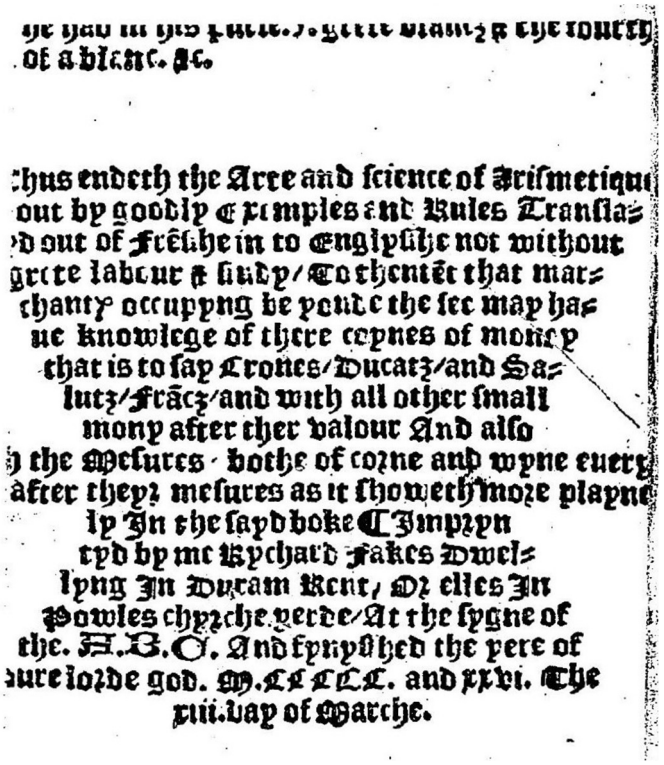
[34] "Because the Bagford Collection preserves the leaf recto side down with paste along the left edge of the recto, the left edges of many lines are obscured but can be reconstructed from first-hand inspection of the leaf" (Williams 2012, p. 165, note 5).

[35] *... he had in his purse .5. grete blancz & the fourth of a blanc &c.* A 'grand blanc' is a French coin worth twelve deniers (according to *Art et science* ≤ 1496, pdf 51, and not five as Williams 2012 writes (p. 170, footnote 22)). Cf. Tropfke 1980, Section 4.2.1.3.3.

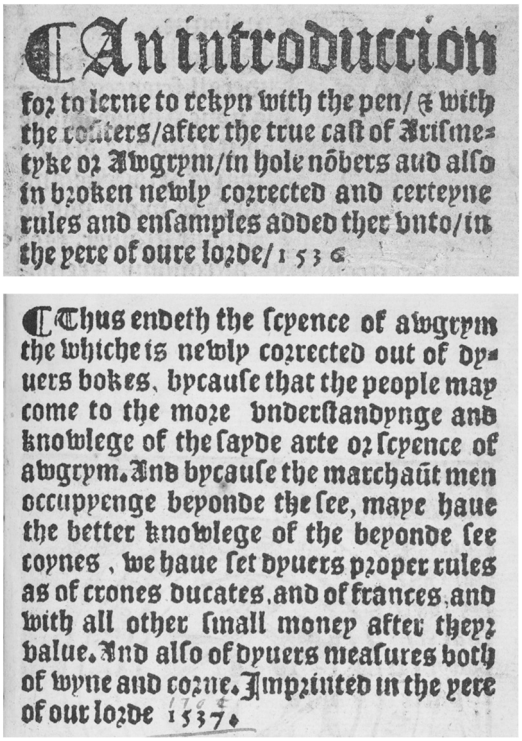
The second printed English arithmetic is *Introduccion* 1537. I will now briefly describe its contents on the basis of Williams 2012, p. 167–183 (including details for the printers on p. 168).[36]

The title terminates with the year 1536, the colophon with 1537.[37]

The prologue starts with *Amonge all the seuen liberall sciences, there is none that maye stande by it selfe but arsmetryke* (sic!) and "immediately establishes arithmetic as a prerequisite for all of the other liberal arts".[38]



Colophon of *Arte and science* 1526-03-13



Title and colophon of *Introduccion* 1537

After the prologue, the book is arranged in 6 not numbered parts (for my numbers of the problems cf. the comparison table (4.2)):[39]

Part [1]: Eight species with the pen for integers: numeration, addition, subtraction, multiplication, division (*partition*), duplication, halving, progression

Part [2]: Seven species (*speche* k.i^{v}) for fractions: numeration, addition, subtraction, multiplication, division, duplication, halving ("mediation"); "uses Dutch terminology throughout" (Williams 2012, p. 181)

Part [3]: Multiplication phrases

Part [4.1]: Regula de tri (*rule of three*) for integers (*whole numbers*), fractions Regula de tri for measures (corn, wine and oil) and weights

[36] Regarding *Introduccion* 1537, I rely on Williams 2012. I did not inspect it by myself as the British Library does not provide a digital copy for free or for reasonable fees. Furthermore, a reprint of the similar *Introduction* 1539 (already discussed in Richeson and Bockstaele 1959/1960) is easily accessible.

[37] The book may have been printed in late March, with the early signatures printed in one year, and later signatures printed in the next (Williams 2012, p. 167).

[38] See Williams 2012, p. 171, for more details.

[39] The structuring (parts and their numbering) follows Bockstaele 1959, p. 124, = Bockstale 1960, p. 319, who introduced it for the structurally similar *Introduction* 1539.

Part [4.2]: Regula societatis (*rule of company*) without and with time
Factors (*rule of company of factors*)
Barter (*rule of changes for to use deceit or fraud*)
Part [4.3]: Many rules and questions: [A1]–[A30] ([A30] "the three saints");
then a "break, bridged in the 1536/7 and 1539 editions with a simple statement: *Here folowe dyuers other proper rules and queſtyons*" (Williams 2012, p. 174)
Part [4.4]: Diverse other proper rules: 6 problems [B1]–[B4], [C1]–[C2]
Part [5]: Five species with counters for integers: numeration, addition, subtraction, multiplication and division
Part [6]: [Rules]
Part [6.1]: [Basic rules]
Regula de tri (*golden rule*, *regula aurea*) for quantities and prices; the section "returns to the rule of three, here renamed 'the golden rule', presented as if it has not been taught before" (Williams 2012, p. 174)
Regula societatis (*rule of company*), regula societatis temporum (*rule of company with time*)
Rule of bartering
Part [6.2]: [Other rules]: [B5]–[B12]

I will continue with comparisons of the earliest printed English arithmetics.
I start with a comparison of the remains of *Arte and science* 1526 with the corresponding parts of *Introduccion* 1537 (the second to the earliest printed English arithmetic):

1. The last problem ("the three saints") of *Arte and science* 1526 appears as last problem in the end of Part [4.3] of *Introduccion* 1537 (see below).
2. The colophon of *Arte and science* 1526 and the colophon of *Introduccion* 1537 (transcription in Williams 2012, p. 167) are similar with one important difference: The first one claims that it is *Translatyd out of Frensche*, the second one that it is *newly corrected out of dyuers bokes* no longer referring to a French source.

Beyond my immediate goals, I will now summarize the examination of differences between *Introduccion* 1537 and *Introduction* 1539[40] in Williams 2012, p. 167–183 (all quotations in this small section refer to Williams's paper).
Both editions "cover roughly the same topics in the same order" (p. 170).
There are only two structural differences:
The multiplication phrases are transposed from the end of Part [1] in *Introduccion* 1537 to a place after the section on fractions (Part [2]) in *Introduction* 1539 (p. 173).
Introduction 1539 terminates with the new Part [6.3] on the simple and double false position method, "a section unique to it, as advertised on its title-page" (p. 174).
The other differences are smaller and do not influence the structure of the book. "The contents of the two editions agree but sparsely in the early pages devoted to the introductions, numeration, addition, subtraction, and multiplication" (p. 171).
The prologue was changed: The "rather 'scholastic' opening" of *Introduccion* 1537 "contrasts with the shorter … 'commercial' opening" of *Introduction* 1539 (p. 171).

[40] Information about later editions can be found in Hoock/Jeannin I/-16 and Williams 2012, p. 168.

Throughout multiplication, *Introduccion* 1537 presents "many more examples and further methods of checking one's work. The 1539 edition only teaches the inversion of operations (e.g., addition to check subtraction) and the common … technique of 'casting out nines'" (p. 172).
Introduccion 1537 "includes a section on" compound amounts with higher and lower units of currency (not "broken numbers" as Williams 2012 writes) "within its treatment of each basic operation, an arrangement also present" in the Dutch tradition (p. 180). "*Introduction* 1539 does not include these paragraphs, in part because it prefers to treat" compound amounts "as 'regular' numbers by handling them through the processes of reduction, which techniques appear in the 1539 edition, but not in the earlier English edition or in any of the Continental sources" (p. 180) – except for *Art et science* ≤ 1496 as we now know.
Introduccion 1537 "includes instruction on special types of multiplication and division: duplation …, triplation …, quadruplation …, mediation …, tertie (division by three), and quadrye (division by four). *Introduction* 1539 ignores all of these (as well as multiplication by squares, a technique that is taught in the 1536/7 edition)" (p. 172). As a consequence, "several narrative examples of duplation and triplation (nails in horseshoes, and two narratives about apples in a garden [e3^{v}–7^{v}]) … were cut by 1539" (p. 183).
Introduction 1539 "next has a section on 'reduction' [species 6] that does not appear in the 1536/7 edition" (p. 172).
"Both editions teach fractions in the same way, considering numeration, addition, subtraction, multiplication, and division, but with the difference that the 1539 edition does not replicate the sections on duplation and mediation of fractions found between subtraction and multiplication in the 1536/7 edition" (p. 173).
"The 1536/7 lesson on fractions uses Dutch terminology throughout" (p. 181) whereas "the vocabulary used for fractions in *Introduction* 1539 is largely Dutch, with a handful of intriguing exceptions confined to the early paragraphs of the section" (p. 180).

4.2 Comparison table

The purpose of the comparison table (stretches always over two opposite pages) is to visualize the development of the structures of the earliest French arithmetics (starting from the left), the earliest Dutch arithmetics (starting from the right) and the earliest English arithmetics (in the middle between French and Dutch).
The structuring (parts and their numbering) of *Introduccion* 1537 and *Introduction* 1539 follows Bockstaele 1959, p. 124, = 1960, p. 319.
The sequence of the lines is oriented towards the sequence of the parts of *Introduction* 1539 that differs from *Introduccion* 1537 only in the position of the multiplication phrases and the supplement of the false position method.
Regarding the English books, problem numbers starting with [A] refer to French sources, [B] to Dutch sources, [C] to additional problems added in the end of *Die maniere* ca. 1510.
Gaps (space lines) in this sequence are set in order to align this sequence with the sequence of other books: duplication, halving ("mediation"), root extraction, one line after [A15], one line after [A25], three lines after [A28], one line after [B3], one after [B7], two lines after [B9], one line after [B11].
Shaded lines show which problems in the Dutch books could be copied from the earliest French book.
My own assumptions, conjectures and comments are enclosed in brackets.
I often use pdf numbers instead of page or leaf numbers as finding a page with the latter can be very time consuming.

Section	**Problem category (Tropfke)**	***Art et science* ≤ 1496**	***Livre de chiffres* 1501**	***Art et science* 1512/19**	***La vraye manière* ca. 1535**
Number of parts		6 pts.	[6] pts.	6 pts.	6 pts.
Begin of the preface		Les philosophes; choses produites par nombre	Pour ce que toutes choses consistent	Proême like *Art et science* ≤ 1496	Proême like *Art et science* ≤ 1496
Part		Pt. 2	Pt. [2]	Pt. 1	Pt. 1
Species with the pen		6 spec. pdf 14	7 spec. pdf 23	9 spec. pdf 12	9 spec. pdf 13
- Num., add., subtr., mult., div.		x	x	x	x
- Reduction		spec. 3	--	--	--
- Duplication		--	spec. 5	spec. 4	spec. 4
- Halving (“mediation”)		--	spec. 4	spec. 5	spec. 5
- Progression		--	--	spec. 8	spec. 8
- Root extraction [comm. = only brief comment]		--	--	[comm.] pdf 17	[comm.] pdf 17
Part		Pt. 3	Pt. [3]	Pt. 3	Pt. 3
5, 6 or 7 rules for fractions (5: num., add., subt., mult., div.) [red. = reduction]		6 spec. (red. 2) pdf 22	6 spec. (red. 2) pdf 49	6 spec. (red. 2) pdf 121	6 spec. (red. 2) pdf 113 pdf 119
Applications of fractions		pdf 27	pdf 60	pdf 131	pdf 122
Part		Pt. 2 end	Pt. [2] end	Pt. 1 end	Pt. 1 end
Multiplication phrases		pdf 21	pdf 48	pdf 99	pdf 91
Part		Pt. 4	Pt. [4]	Pt. 4	Pt. 4
Rule of three (7 or 4 basic rules); measures (3 rules); weights		7+3 pdf 29	7+3 pdf 65	4+3 pdf 136	4+3 pdf 126
- Measures, rule nr. 3: Inverse rule of three (loaf of penny)		pdf 34	pdf 76	pdf 144	pdf 134
Part		Pt. 5	Pt. [5]	Pt. 5	Pt. 5
Rule (nr. 1) of company without time	4.1.9 Companies	pdf 36	pdf 81	pdf 148	pdf 138
Rules (nr. 2–4, except 1490) of company with time	4.1.9 Companies	pdf 37	pdf 85	pdf 150	pdf 140

***Art et science* ca. 1545**	***Arte and science* 1526**	***Intro-duccion* 1537**	***Intro-duction* 1539**	***La manière* 1529 [BNF]**	***Die maniere* ca. 1510**	***Die maniere* 1508**
6 pts.		[6] pts.	[6] pts.	[4] pts.	[4] pts.	[3] pts.
Proême like *Art et science* ≤ 1496		Amonge all the seuen liberall sciences	To the reader: That arte and feate	Pour ce que entre les sept sciences liberalles	Want onder die seuen vrie consten	Want onder die seuen vrie consten
Pt. 1		Pt. [1]	Pt. [1]	Pt. [1]	Pt. [1]	Pt. [1]
9 spec. pdf 13		8 spec.	7 spec. fol. Aii^{v}	7 spec. pdf 5	7 spec. pdf 5	7 spec. p. 5
x		x	x	x	x	x
--		--	spec. 6	--	--	--
spec. 4		x	--	spec. 5	spec. 5	spec. 5
spec. 5		x	--	spec. 4	spec. 4	spec. 4
spec. 8		spec. 8	spec. 7	--	--	--
[comm.] pdf 17		--	--	--	--	--
Pt. 3		Pt. [2]	Pt. [2]	Pt. [2]	Pt. [2]	Pt. [2]
6 spec. (red. 2) pdf 108		7 spec. (duplic., halving, no red.)	5 spec. (no red.) fol. Fi^{v}	species 8-14 (no red.) pdf 44	species 8-14 (no red.) pdf 24	species 8-14 (no red.) p. 24
pdf 117		--	--	--	--	--
Pt. 1 end		Pt. [1] end	Pt. [3]			
pdf 87		fol. L1^{r}	fol. Fv^{r}	--	--	title page
Pt. 4		Pt. [4.1]	Pt. [4.1]			
4+3 pdf 121		4+3	4+3 fol. Gi^{r}	mesures du vin last page pdf 102	--	--
pdf 127		x	fol. Gv^{r}	--	--	--
Pt. 5		Pt. [4.2]	Pt. [4.2]			
pdf 131		x	fol. Gvii^{r}	--	--	--
pdf 133		x	fol. Gviii^{r}	--	--	--

Section	**Problem category (Tropfke)**	***Art et science* ≤ 1496**	***Livre de chiffres* 1501**	***Art et science* 1512/19**	***La vraye manière* ca. 1535**
Companies with factors (3 rules)	4.1.10 Factors	pdf 40	pdf 91	pdf 156	pdf 145
Rule of bartering(3 problems)	4.1.2 Barter	pdf 42	pdf 96	pdf 161	pdf 149
Part		Pt. 6	Pt. [6]	Pt. 6	Pt. 6
Collect and tallage: proportional distribution	4.1.9 Companies: Special case	1 pdf 43	1 pdf 99	1 pdf 164	1 pdf 152
Three mills: different performances	4.2.1.2.5 Mills	2 pdf 45	2 pdf 102	2 pdf 167	2 pdf 154
Shepherd: proportional distribution	4.1.9 Companies	3 pdf 46	3 pdf 103	3 pdf 169	3 pdf 156
Vessel with three fountains or holes: different performances	4.2.1.2.1 Cisterns	4 pdf 47	4 pdf 105	4 pdf 171	4 pdf 157
To throw Saracens in the sea	4.2.7.3 "Joseph-Spiel"	5 pdf 47	5 pdf 107	5 pdf 172	--
Testament (son daughter twins) [similar to *Die maniere* ca. 1510 [7]]	4.2.7.4 Twin heritage	6 pdf 48	6 pdf 108	6 pdf 174	5 pdf 159
Building - place: area, houses	4.2.3 Geometry	7 pdf 49	7 pdf 109	7 pdf 175	6 pdf 160
Building - walls: volume, price	4.2.3 Geometry	8 pdf 49	8 pdf 110	8 pdf 176	7 pdf 160
Building - covering: area	4.2.3 Geometry	9 pdf 49	9 pdf 111	[9 error pdf]	8 pdf 161
Apple garden: nest problem	4.2.1.3.2 Apple garden	10 pdf 50	10 pdf 111	10 pdf 178	--
Ladder with 100 steps	4.2.4.1 Arithmetic progressions	--	--	11 pdf 179	9 pdf 162
Movement in the same direction with constant vs increasing distances per day (thief) [similar to *Introd.* 1539 [B6]]	4.2.1.4 Movement: Pursuit	11 pdf 50	11 pdf 112	--	--
Movement in the same direction with constant vs increasing distances per day (two men) [like *Introduction* 1539 [B6]]	4.2.1.4 Movement: Pursuit	--	--	12 pdf 180	10 pdf 163

***Art et science* ca. 1545**	***Arte and science* 1526**	***Intro-duccion* 1537**	***Intro-duction* 1539**	***La manière* 1529 [BNF]**	***Die maniere* ca. 1510**	***Die maniere* 1508**
pdf 138		x	fol. Hii^{v}	--	--	--
pdf 142		fol. N4^{v}	fol. Hv^{r}	--	--	--
Pt. 6		Pt. [4.3]	Pt. [4.3]			
1 pdf 145		[A1]	[A1] fol. Hvi^{v}	--	--	--
2 pdf 147		[A2]	[A2] fol. Hviii^{r}	--	--	--
3 pdf 149		[A3]	[A3] fol. Ji^{r}	--	--	--
4 pdf 151		[A4]	[A4] fol. Ji^{v}	--	--	--
5 pdf 152		[A5]	[A5] fol. Jii^{v}	--	--	--
6 pdf 153		[A6]	[A6] fol. Jiii^{r}	--	--	--
7 pdf 155		[A7]	[A7] fol. Jiv^{r}	--	--	--
8 pdf 155		[A8]	[A8] fol. Jiv^{r}	--	--	--
9 pdf 156		[A9]	[A9] fol. Jiv^{v}	--	--	--
10 pdf 156		[A10]	[A10] fol. Jiv^{v}	--	--	--
11 pdf 157		[A11]	[A11] fol. Jv^{r}	--	--	--
--		--	--	--	--	--
12 pdf 158		[A12]	[A12] fol. Jv^{v}	--	--	--

Section	**Problem category (Tropfke)**	***Art et science* ≤ 1496**	***Livre de chiffres* 1501**	***Art et science* 1512/19**	***La vraye manière* ca. 1535**
Three women and apples for the market	4.2.2.6 100 birds: Special case	13 pdf 51	13 pdf 114	13 pdf 182	--
Batch of three metals: alloy [like *Introduction* 1539 [B10]]	4.1.14.1, 4.1.9 Mixture	14 pdf 52	14 pdf 115	14 pdf 183	11 pdf 164
Bell: alloy	4.1.14.1, 4.1.9 Mixture	15 pdf 52	15 pdf 116	15 pdf 184	--
Movement in the same direction with margin and constant distances per day	4.2.1.4 Movement: Pursuit	16 pdf 53	16 pdf 117	[16 not]	--
Gold coins to silver coins: equal quantities of different silver coins [like *Introduction* 1539 [B11]]	4.2.1.5.1 Regula aequalitatis	17 pdf 53	17 pdf 118	17 [16] pdf 185	12 pdf 165
Cloth of divers colours	4.2.1.1.3 Spear in water	18 pdf 54	18 pdf 119	18 [17] pdf 186	13 pdf 166
Spices: equal quantities	4.2.1.5.1 Regula aequalitatis	19 pdf 54	19 pdf 120	19 [18] pdf 186	14 pdf 166
Woman and eggs: remainder problem [like *Die maniere* ca. 1510 [4]]	4.2.5.2 Remainder: Eggs	20 pdf 54	20 pdf 121	19 pdf 187	15 pdf 167
Money forgotten with a changer: nest problem [similar to *Introduction* 1539 [B9]]	4.2.1.3.5 Unknown heritage	21 pdf 55	21 pdf 122	20 pdf 189	16 pdf 168
Age (time): sum of fractions	4.2.1.1.2 Age	22 pdf 56	22 pdf 123	21 pdf 190	17 pdf 170
Divide a distribution: proportional	4.1.9 Companies: Special case	23 pdf 56	23 pdf 124	22 pdf 191	18 pdf 170
Spear in water	4.2.1.1.3 Spear in water	24 pdf 56	24 pdf 125	23 pdf 192	19 pdf 171
Movement in opposite directions with different distances per day (two men go against each other) [like *Introduction* 1539 [B7]]	4.2.1.4 Movement: Encounter	25 pdf 57	25 pdf 125	24 pdf 192	20 pdf 172
Cat on a tree	4.2.1.4 Movement: To and fro	26 pdf 57	26 pdf 126	25 pdf 193	21 pdf 172

Art et science **ca. 1545**	***Arte and science*** **1526**	***Intro-duccion*** **1537**	***Intro-duction*** **1539**	***La manière*** **1529 [BNF]**	***Die maniere*** **ca. 1510**	***Die maniere*** **1508**
13 pdf 160		[A13]	[A13] fol. Jvi^{v}	--	--	--
14 pdf 161		[A14]	[A14] Jvii^{r}	--	--	--
15 pdf 162		[A15]	[A15] fol. Jvii^{v}	--	--	--
--		--	--	--	--	--
16 pdf 163		[A16]	[A16] fol. Jviii^{v}	--	--	--
17 pdf 164		[A17]	[A17] fol. Jviii^{v}	--	--	--
18 pdf 165		[A18]	[A18] fol. Ki^{r}	--	--	--
19 pdf 165		[A19]	[A19] fol. Ki^{v}	cf. Pt. [3.2], probl. [4]	cf. Pt. [3.2], probl. [4]	cf. Pt. [3.2], probl. [4]
20 pdf 167		[A20]	[A20] fol. Kii^{v}	--	--	--
21 pdf 168		[A21]	[A21] fol. Kii^{v}	--	--	--
22 pdf 169		[A22]	[A22] fol. Kiii^{r}	--	--	--
23 pdf 170		[A23]	[A23] fol. Kiii^{v}	--	--	--
24 pdf 170		[A24] fol. P3^{r}	[A24] fol. Kiv^{r}	--	--	--
25 pdf 171		[A25]	[A25] fol. Kiv^{r}	--	--	--

Section	**Problem category (Tropfke)**	***Art et science* ≤ 1496**	***Livre de chiffres* 1501**	***Art et science* 1512/19**	***La vraye manière* ca. 1535**
Ship with two sails: different performances	4.2.1.2.3 Sails	27 pdf 58	27 pdf 127	--	--
Disciples and hoste: regula de tri	4.1.1 Buy and sell	28 pdf 58	--	26 pdf 194	22 pdf 173
The pilgrims' drink bill	4.2.2.6 Regula caecis	29 pdf 58	28 pdf 128	27 pdf 194	23 pdf 173
The chanter's rent	4.2.2.6 Regula caecis	30 pdf 58	29 pdf 128	28 pdf 195	24 pdf 174
Guess the number of pieces of silver in a purse $5(n + n)$	4.2.6 Number guessing	31 pdf 59	[30 not] 31 [30] pdf 128	--	--
Guess a number $n/2$	4.2.6 Number guessing	32 pdf 59	32 [31] pdf 129	--	--
Three men find a purse: one unknown	4.2.1.1.3 Spear in water	33 pdf 59	--	--	--
Guess the number of silver pieces in your fellow's right hand (10 pieces remaining)	4.2.6 Number guessing	34 pdf 59	--	29 pdf 195	25 pdf 174
"The three saints": nest problem	4.2.1.3.3 Si quis intrat monasterium	12 pdf 51	12 pdf 113	30 pdf 196	26 pdf 175
Part					
Lord and servant	4.2.1.5.3 Interrupted employment				
Three young men play dice: equal amounts in the end	4.2.6.6 Three players				
Weight-price relations: 46 and 63 pounds saffron (rule of three)	4.1.1 Buy and sell				
Weight-price relations: 165 and 22 pounds alum (rule of three)	4.1.1 Buy and sell				
Batch of pepper (unknown weight)	4.2.1.5.2 Too much – too little				

Art et science **ca. 1545**	***Arte and science*** **1526**	***Intro-duccion*** **1537**	***Intro-duction*** **1539**	***La manière*** **1529 [BNF]**	***Die maniere*** **ca. 1510**	***Die maniere*** **1508**
--		--	--	--	--	--
26 pdf 172		[A26]	[A26] fol. Kiv^{v}	--	--	--
27 pdf 172		[A27]	[A27] fol. Kiv^{v}	--	--	--
28 pdf 173		[A28]	[A28] fol. Kv^{r}	--	--	--
--		--	--	--	--	--
--		--	--	--	--	--
--		--	--	--	--	--
29 pdf 173		[A29]	[A29] fol. Kv^{r}	--	--	--
30 pdf 174	last problem	[A30] fol. P4^{v}-5^{r}	[A30] fol. Kv^{v}	--	--	--
		Pt. [4.4]	Pt. [4.4]	Pt. [3.2]	Pt. [3.2]	Pt. [3.2]
		[B1]	[B1] fol. Kvi^{r}	[13] pdf 79	[13] pdf 43	[13] p. 89
		[B2]	[B2] fol. Kvi^{v}	[14] pdf 80	[14] pdf 43	[14] p. 90
		[C1]	[C1] fol. Kvii^{r}	Part [4] probl. [3] pdf 99	Part [4] probl. [3] pdf 53	--
		[C2]	[C2] fol. Kvii^{v} like Mvii^{v}	Part [4] probl. [4] pdf 100 last page	Part [4] probl. [4] pdf 54 last page	--
		[B3]	[B3] fol. Kviii^{r}	[15] pdf 81	[15] pdf 44	[15] p. 91

Section	**Problem category (Tropfke)**	***Art et science* ≤ 1496**	***Livre de chiffres* 1501**	***Art et science* 1512/19**	***La vraye manière* ca. 1535**
Almonds (unknown money)	4.2.1.5.2 Too much – too little				
A drunkard, his wife and a barrel of beer: different performances	4.2.1.2.6 Drunkards				
Part		Pt. 1	Pt. [1]	Pt. 2	Pt. 2
Species with counters (4 figures for numeratio, reductio [= red.], multiplication, division)		6 spec. (red. 3) pdf 5	6 spec. (red. 3) pdf 6	6 spec. (red. 3) pdf 103	6 spec. (red. 3) (5 fig.) pdf 95
Four trailing problems [1]: pins (spelden) for 100 sh. [2]: dice (teerlingen) for 100 fl. [3]: saffron (see [C1]) [4]: alum (see [C2])					
Part					
Rule of three with 7 cases: 1 in the 1st / 3rd place mixed fraction in place 1 / 3 / 2 mixed fractions in places 2 / 3 mixed fractions in places 1 / 2					
Rule of company without time	4.1.9 Companies				
Rule of company with time (1 case)	4.1.9 Companies				
Rule of bartering	4.1.2 Barter				
Part					
Hare and greyhound	4.2.1.4 Movement: Pursuit				
Movement in the same direction with constant vs increasing distances per day (two fellows) [like *Introduction* 1539 [A12]]	4.2.1.4 Movement: Pursuit				
Movement in opposite directions with different distances per day [like *Introduction* 1539 [A24]]	4.2.1.4 Movement: Encounter				

***Art et science* ca. 1545**	***Arte and science* 1526**	***Intro-duccion* 1537**	***Intro-duction* 1539**	***La manière* 1529 [BNF]**	***Die maniere* ca. 1510**	***Die maniere* 1508**
		--	--	[16] pdf 82	[16] pdf 44	[16] p. 92
		[B4]	[B4] fol. Kviii^{v}	[17] pdf 83	[17] pdf 45	[17] p. 94
Pt. 2		Pt. [5]	Pt. [5]	Pt. [4]	Pt. [4]	
6 spec. (red. 3) pdf 93	--	5 spec. (no red.) (12 fig.)	5 spec. (no red.) (12 fig.) fol. Li^{r}	5 spec. (no red.) (6 fig.) pdf 84	5 spec. (no red.) (6 fig.) pdf 45	--
		see [C1] and [C2]	see [C1] and [C2]	pdf 99	pdf 52	--
		Pt. [6.1]	Pt. [6.1]	Pt. [3.1]	Pt. [3.1]	Pt. [3.1]
		x	fol. Mvii^{r}	pdf 53	pdf 29	p. 58
		x	fol. Niii^{r}	pdf 62	pdf 34	p. 69
		x	fol. Niii^{v}	pdf 63	pdf 35	p. 71
		x	fol. Niv^{r}	pdf 64	pdf 36	p. 72
		Pt. [6.2]	Pt. [6.2]	Pt. [3.2]	Pt. [3.2]	Pt. [3.2]
		[B5]	[B5] fol. Niv^{v}	[1] pdf 65	[1] pdf 35	[1] p. 73
		[B6]	[B6] fol. Nv^{r}	[2] pdf 66	[2] pdf 36	[2] p. 74
		[B7]	[B7] fol. Nv^{v}	[3] pdf 67	[3] pdf 37	[3] p. 75

Section	**Problem category (Tropfke)**	***Art et science* ≤ 1496**	***Livre de chiffres* 1501**	***Art et science* 1512/19**	***La vraye manière* ca. 1535**
Woman and eggs: remainder problem [like *Introduction* 1539 [A19], *Art et science* ≤ 1496, 20]	4.2.5.2 Remainder: Eggs				
Working and playing	4.2.1.5.4 Lazy worker				
Testament: nest problem [similar to *Introduction* 1539 [A20]]	4.2.1.3.5 Unknown heritage				
Testament (one son and two daughters triplet) [similar to *Introduction* 1539 [A6]]	4.2.7.4 Twin heritage				
Three weights: two cups and a lid	4.2.2.5 Cups and lid				
Goldsmith: alloy [like *Introduction* 1539 [A14]]	4.1.14.1, 4.1.9 Mixture				
Gold coins to silver coins: equal quantities of different silver coins [like *Introduction* 1539 [A16]]	4.2.1.5.1 Regula aequalitatis				
Ship purchase: proportional distribution (sum > 1)	4.1.9 Companies				
Four carpenters build a house: different performances	4.2.1.2.2 House building				
Colophon St. Albans 1537					
Part					
Simple and double false position method					
Colophon London 1539					

Art et science **ca. 1545**	***Arte and science*** **1526**	***Intro-duccion*** **1537**	***Intro-duction*** **1539**	***La manière*** **1529 [BNF]**	***Die maniere*** **ca. 1510**	***Die maniere*** **1508**
		cf. [A19]	cf. [A19]	[4] pdf 68	[4] pdf 37	[4] p. 76
		[B8]	[B8] fol. Nvi^{r}	[5] pdf 69	[5] pdf 38	[5] p. 77
		[B9]	[B9] fol. Nvi^{v}	[6] pdf 70	[6] pdf 38	[6] p. 79
		--	--	[7] pdf 72	[7] pdf 39	[7] p. 80
		--	--	[8] pdf 73	[8] pdf 40	[8] p. 82
		[B10]	[B10] fol. Nvii^{r}	[9] pdf 74	[9] pdf 40	[9] p. 83
		[B11]	[B11] fol. Nvii^{v}	[10] pdf 75	[10] pdf 41	[10] p. 84
		--	--	[11] pdf 76	[11] pdf 41	[11] p. 85
		[B12]	[B12] fol. Nviii^{r}	[12] pdf 78	[12] pdf 42	[12] p. 87
		fol. S8^{r}	--			
Pt. [6.3]						
		--	fol. Nviii^{v}	--	--	--
		--	fol. Ovii^{v}	--	--	--

4.3 The French and Dutch sources of the English arithmetics

The remains of *Arte and science* 1526 are so small that a sustainable result regarding its French source(s) cannot be based on them. Nevertheless, one quickly figures out two correspondences between it and *Art et science* 1512/19: the similarity of the title and the same terminating problem[41]. It is amazing, but this short consideration already delivers the – as will immediately turn out – correct French source if we assume a very broad similarity between *Arte and science* 1526 and *Introduccion* 1537.

So, let me extend the search for possible French sources to *Introduccion* 1537 that survived completely and therefore allows a broader comparison. In spite of uncertainties in the dating of the earliest French arithmetics, one can state that there were only seven such before 1537 (cf. 2.1). Four of them can be excluded due to their different structure (cf. 2.1). Regarding the remaining three, the comparison table (4.2) leads to a striking result: The structures of *Introducction* 1537, Parts [1] through [4.3] (terminating with the problem of "the three saints"), and *Art et science* 1512/19 completely coincide. I included more possible French sources, but still arrive at the same result as Williams 2012 (p. 182).

At the end of Part [4.3] of *Introduccion* 1537, there is a cut where the translation from *Die maniere* ca. 1510 starts, a fact that already Bockstaele 1959/60 had found. There is nothing new to say about Parts [4.4] through [6.2] that possess a similar structure as Parts [3.1] through [4] of *Die maniere* ca. 1510 respectively its French translation *La manière* 1529. The coincidence is not so striking as in the case of *Art et science* 1512/19. So far my remarks with regard to the structures.

The supplement (Part [6.3]) of *Introduction* 1539 is on the false position method. I was not able to find its source.[42]

Differences in details are normal and due to the redactor's considerations and the interference with possible other sources. I follow Bockstaele (1960, p. 319–321) who compares *Introduction* 1539 to *Die maniere* ca. 1510 and *La manière* 1535. His comparison is still valuable as the latter is very similar to all three of its predecessors and *Introduction* 1539 is similar to *Introduccion* 1537.

Part [1] shows some coincidences with both Dutch and French sources.

Part [2] about fractions needs a larger comment.

Art et science 1512/19 contains mistakes regarding multiplication and division of fractions (cf. 2.3). Regrettably, we do not know anything about this subject in *Arte and science* 1526 as it did not survive. The part of *Introduccion* 1537 on fractions is a translation of Part [2] of the Dutch source *Die maniere* ca. 1510; it only used Dutch based terms for fractions, the originally Dutch words *teller* and *number* (Dutch *noemer*) instead of the English *numerator* and *denominator*.[43] This is certainly due to the mistakes in the French source that the English redactor recognized and wanted to avoid. The situation in the corresponding part of *Introduction* 1539 is more complicated: In the beginning, "the text refers to a numerator and a denominator … Subsequently the text reverts to Dutch vocabulary".[44] Williams saw this discrepancy, but could not explain it.

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[41] "The three saints"; cf. comparison table (4.2).

[42] Neither from French nor from Dutch; also *Juan de Ortega* 1512, 1515, *Santcliment* 1482, *Pellos* 1492 and *Gielis van den Hoeck*e 1537 are different.

[43] Cf. Williams 2012, p. 175 and p.180–181; Bockstaele 1959, p. 125–126.

[44] Cf. Williams 2012, p.180–181.

Obviously, the redactor of *Introduction* 1539 returned to the original French source (in contrast to *Introduccion* 1537), but used it for the introductory aspects of fractions only. As soon as the French text arrives at the mistakes in multiplication and division, however, the redactor jumped to the Dutch source.
Part [4.1] through [4.3] are translations of Part 4 through 6 of the French source.
In Part [4.4], the problems [B1]–[B4] and [C1]–[C2] are translated from the second half of Part [3.2] of *Die maniere* ca. 1510.
Part [5] is taken from Part [4] of the Dutch source and enriched with examples.
Part [6.1] has its origin in Part [3.1] of the Dutch source and [6.2] ([B5]–[B12]) in the first half of [3.2], but some examples are omitted.

Hopefully, one day in the future, a complete copy of *Arte and science* 1526 will enter the public domain (as *Introduccion* 1537 – which was also thought to survive as a single leaf only – did in 2005) and will help us to answer further open questions regarding the earliest printed English arithmetics.

5 References

5.1 Primary sources – Bibliography table in temporal sequence

Shaded lines mark books outside the traditions that immediately influenced the English arithmetics. Digital copies are not publically available for all of the books.

(Author:) Title	Year, place, printer, pages	Estimation of the year of publication	Library, catalog
De arte numerandi sive arismetice summa quadripartite	ca. 1490 Paris Antoine Caillaut or Louis Martineau 40 p.	Caillaut active: 1482-1506 (cnp02226310 CERL Thes.) Martineau active: 1482-1498 (cnp02244059 CERL Thes.)	Freiburg Univ. London BritLib UCatInc 0266410N ISTC ia01136100
Art et science d'arithmetique par exemples et practique	≤ 1496 Paris Michel Toulouse 112 p.	Toulouse active: 1492-1505 (cnp02243351 CERL Thes.) Paris ENSBA: ≤ 1496 [acc. to the address (Coq, Dominique 2012, 45)]	Paris ENSBA (Masson 0603) UCatInc 0267250N ISTC ia01140600
Livre de chiffres et de getz nouvelle-ment imprimé	1501-02-27 Lyon Pierre Mareschal, Barnabe Chaussard 127 p.		Augsburg SB München BSB
Die maniere om te leeren cyffren	1508-09-09 Brussel Thomas van der Noot 96 p.		Brussel KBR (II 53.133 A) Hoock/Jeannin I/-20.1
Die maniere om te leeren cyffren ende rekenen metter pennen end metten penningen	ca. 1510 Antwerpen Willem Vorsterman 104 p.	Vorsterman active: 1504-1543 (cni00035224 CERL Thes.) Hoock/Jeannin: ca. 1510 Amsterdam Univ.: 1510?	Amsterdam Univ. (Ned. Inc. 293) Hoock/Jeannin I/-20.2

(Author:) Title	Year, place, printer, pages	Estimation of the year of publication	Library, catalog
Arismétique corrigée et imprimé à Paris	ca. 1512/15 Paris Guillaume Nyverd 127 p.	Nyverd active: 1500-1519 (cnp01332278 CERL Thes.) Hoock/Jeannin: 1512/1515 Harvard Univ.: ca. 1515	Harvard Univ. - Houghton Lib. (FC 5 A 100 515 a) Hoock/Jeannin I/-3.1
L'art et science de arismetique: moult vtille et proffitable a toutes gens	ca. 1512/19 Paris Veuve feu [deceased] Jean Trepperel and Jean Jehanot 192 p.	Widow Trepperel and Jehanot together active: 1512-1519 (cni00011737, cni00007397 CERL Thes.) Columbia Univ.: 1510/20	Columbia Univ. - Plimpton Lib. (511 1520 Ar75)
Juan de Ortega, Claude Platin: *Oeuvre tressubtille et profitable* [based on Ortega: *Conpusicion,* Lyon 1512]	1515 Lyon Étienne Baland 332 p.		Paris BibNatFr (Rés. p.V. 369) Hoock/Jeannin I/O6.3
Estienne de La Roche: *L'arismétique*	1520 Lyon Guillaume Huyon 460 p.		Paris BibNatFr (Rés. V. 899) Hoock/Jeannin I/L5.1
Arte and science of arismetique	1526 London Richard Faques ? p.		London BritLib (Bagford 178) Digital: Early English Books Online EEBO
La manière pour apprendre a cyfrer	1529 Antwerpen Martin Lempereur for Willem Vorsterman ca. 105 p.		Paris BibNatFr (incomplete) (Rés. p.V. 338) Harvard Univ. - Houghton Lib. (FC5 A100 529m) [different editions (Williams, p. 177)] Hoock/Jeannin I/-20.4
La vraye manière; colophon: *Art et science*	ca. 1535 Lyon Claude Veycellier 168 p.	Veycellier active 1528-1536 (cnp01393948 CERL Thes.) Hoock/Jeannin: ca. 1535 London BritLib: ca. 1535	London BritLib (8506.aa.29) dig. Google Hoock/Jeannin I/-20.5
Godefroyd Gomparst: *Un livre de compte*	ca. 1535 Lyon Simon Cock 288 p.		London BritLib (C.54.a.1) Hoock/J. I/G12.1
Gielis van den Hoocke: *Een sonderlinghe boeck*	1537 Antwerpen Simon Cock 360 p.		2nd edition 1545: Amsterdam EHB (Econom.-Hist. Bib.) Hoock/Jeannin I/V.3

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Introduccion for to lerne to rekyn	1537 St. Albans John Herford for Richard Stevenage 288 p.		London BritLib (C.194.a.450) Hoock/Jeannin I/-16.1
Introduction for to lerne to reckon	1539 London Nicolas Bourman 221 p.		Oxford BodLib (STC. 14118) Hoock/Jeannin I/-16.2
Art et science de arismetique moult utille et proffitable a toutes gens	ca. 1545 Paris Pierre Sergent 168 p.	Sergent active 1532-1547 (cnp01346163 CERL Thes.) Hoock/Jeannin: 1520 [error] Paris BibNatFr: ca. 1547 [l'état de la marque]	Paris BibNatFr (Rés. p.V. 337) Hoock/Jeannin I/-6.1

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